Using Continuous Wavelet Transform to Construct the Dispersion Image for Soil Layers
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Reference

ABSTRACT
This study used a time-frequency domain analysis for estimating the dispersion curve of a Rayleigh wave by using two receivers. The signals were first transformed using continuous wavelet transform. A similar slant stack procedure was used to analyze the wavelet transform signals and extract a dispersion image. This method is advantageous because it requires no empirical judgment in phase unwrapping and few receivers. To examine the applicability of the method for evaluating the dispersion curve for soil layers with lateral heterogeneity, three synthetic examples and an experience example were investigated. In these examples, numerical simulations of the surface wave seismic test were performed using the finite difference FLAC code. The results revealed that the estimates of the surface wave dispersion curve, obtained using the method, coincide with those of the theoretical values. A high-resolution dispersion image is generated by increasing the spacing of receivers. The method is applicable for extracting a dispersion image for lateral heterogeneous soil layers.

Keywords
dispersion curve, continuous wavelet transform, shear wave velocity, Rayleigh wave, lateral heterogeneity

Introduction
The shear wave velocity profile is a crucial part of seismic risk characterization. The shear wave velocity profile is usually obtained using the surface wave seismic test because the surface wave is the predominant wave on the ground surface; additionally, it is the most easily measured wave. The dispersion characteristic of surface waves in a layered soil stratum is a phenomenon involving...
The dispersion curve of the surface wave represents the relationship between wave velocity and frequency. The dispersion curve obtained from a surface wave seismic test can be used in an inversion algorithm to estimate the shear wave velocity profile. One such test is by using the spectral analysis of surface waves (SASW) method, which involves installing two receivers on the ground surface and measuring the phase velocity in the frequency domain from the phase shifts between the two receivers. The SASW method has been implemented successfully in geotechnical and pavement engineering since the method was introduced by Nazarian et al. [1]. However, the signal-processing procedure is difficult and requires an empirical judgment for constructing the dispersion curve from the surface wave signals. Furthermore, the dispersion curve may be constructed incorrectly because of the spurious 360° cycles during phase unwrapping [2]. Some criteria regarding the feasible configuration of receiver to perform SASW were suggested in Refs 3–6. Recently, some studies by using the SASW method were reported in Refs 7–9.

To reduce the difficulty of evaluating the dispersion curve when using the SASW method, the multichannel analysis of surface waves (MASW) technique was developed. In the method, which involves increasing a number of receivers along a survey line, a spatial resolution is acquired using a substantial number of geophones and a shot gather is measured using a multichannel recording system. The particle motion velocity time histories are transformed and analyzed to extract a dispersion image in the wave velocity–frequency domain. The dispersion curve is designated as the peak of the dispersion image.

Thorson and Claerbout [10] proposed tau-p transform or the so-called slant stack, to perform the velocity spectral analysis. This transformation takes multiple seismograms, which takes amplitudes related to distance and time (x-t), and converts it into amplitudes related to slowness and intercept time. In this study, the method was applied to extract a dispersion image from time–frequency distribution of seismic data. The MASW method can be difficult to manipulate in an urban area because it requires a substantial number of receivers with a wide receiver spread.

Time–frequency analysis can be performed with continuous wavelet transform (CWT) to understand the transient changes in spectra. The wavelet transform uses a size-adjustable window, ensuring that the duration of the window is short at high frequencies and long at low frequencies. Therefore, wavelet transform has a high-time resolution at high frequencies, and has a high-frequency resolution at low frequencies [11]. The transformed figure shows the distribution of the transform signal at the time-frequency plane, enabling the simultaneous acquisition of the signal characteristics in the time domain and frequency domain. In the literature about the application of CWT on solution for seismic wave decomposing and denoising (e.g., Refs 12–16), Kim and Park [12] used harmonic wavelet transform to determine the dispersion curve. They concluded that the proposed method is less affected by noise and near field effect than the phase unwrapping method. Park and Joh [13] applied the harmonic wavelet for phase spectrum assessment to reduce background noise effects. Kim and Kwan [14] proposed a new technique to compute the Rayleigh wave velocity using a principal wavelet-component analysis to exclude noise and reflection waves. Rosyidi and Taha [15] utilized CWT based on Gaussian derivative wavelet to decompose seismic signals and filter noise on SASW tests. Golestani et al. [16] used CWT to localize the recorded seismic signals and applied discrete wavelet transform to reduce noise on SASW tests.

The present study performed a time–frequency analysis to extract the dispersion image of the surface wave for lateral heterogeneous soil layers. This method, based on CWT, supplies dependable information on the surface wave dispersion curve, and requires only two receivers. Therefore, this method does not necessitate the use of numerous geophones and requires little time. To transform a signal generated from surface wave seismic testing, a value of 0.8125 Hz was set as the center frequency of the real-valued Morlet wavelet function in the Matlab wavelet toolbox. A similar slant stack procedure was then used to analyze the wavelet transform signals and extract a dispersion image. Three synthetic examples and an experimental example of surface wave seismic test were examined in this paper to estimate the applicability of the time–frequency analysis for soil layers with lateral heterogeneity.

**Method**

**CONTINUOUS WAVELET TRANSFORM**

To transform an original signal into the form of dilation parameter and time, a wavelet function $\psi(t)$ is used to perform the translation and dilation processes. Frequency is inversely proportional to the dilation parameter; thus, the relationship between the frequency and dilation parameter is defined as follows [17]:

$$f = \frac{f_0}{a\Delta t}$$  \hspace{1cm} (1)

where:

- $f_0$ = the center frequency of a wavelet function,
- $a$ = dilation parameter, and
- $\Delta t$ = the sample period.

To transform a signal generated from surface wave testing, the value of 0.8125 Hz was set as the center frequency of the real-valued Morlet wavelet function in the MATLAB wavelet toolbox. The real-valued Morlet wavelet function is defined as follows [17]:

$$\psi(t) = C \cos(5t)e^{-t^2/2}$$
where:

\[ C = \text{a constant used for normalization to achieve reconstruction.} \]

A surface wave seismic test was simulated using the numerical software FLAC \[18\]. The surface wave test in the two-layer soil model, the thickness of the upper soil is equal to 5 m. Fig. 1a is the vertical-component seismogram of particle motion velocity at a source-to-receiver offset of 10 m. Fig. 1a shows that the arrival time of the Rayleigh wave is approximately 0.05 s. If the particle motion velocity time history was transformed into the time–frequency domain with CWT, the frequency and time content of the seismic signal \( w(t,f) \) was obtained simultaneously, as shown in Fig. 1b. The figure shows that the range of 0.05–0.07 s and 100–250 Hz clearly indicates a strong seismic energy. The temporal information in each signal frequency can be obtained from Fig. 1b. This procedure is used to extract the dispersion image in the following paragraphs.

**FIG. 1** Synthetic seismic record at a source-to-receiver offset of 10 m during seismic testing for the thickness of the upper layer soil = 5 m: (a) seismogram and (b) time–frequency distribution by continuous wavelet transform.

**DISPERSION IMAGE EXTRACTED BY THE CWT AND SLANT STACK**

Because surface waves possess a maximum energy density, the peaks of the wavelet transformed signals may be generated by surface waves. This characteristic is beneficial in extracting the surface wave component from a time–frequency distribution obtained using CWT. The wave travel time between the two receivers is equal to the receiver spacing divided by wave velocity. The arrival time of the seismic wave to the first and second receivers is assumed to be \( t_1 \) and \( t_1 + \text{wave travel time} \), respectively. The spectrum value for a certain frequency and wave velocity was firstly calculated as the product values of the wavelet transformation signals corresponding to the aforementioned arrival time of the seismic wave for the two receivers. The spectrum value was then calculated by summing the products at all assuming \( t_1 \). By repeating these procedures for all wave velocities and frequencies, the dispersion image was obtained.

The main process for deriving dispersion curves in this study involved the following steps:

**Step 1:** Time series data from a pair of receivers are transformed into the signal of the time–frequency domain by using CWT.

**Step 2:** At a certain frequency, the two transformed signals are normalized at the maximum transformed values of each receiver.

**Step 3:** The slant stack summation is applied to the two temporal plots. For a certain wave velocity, the summed value is assigned to a point in the frequency–wave velocity plane. This step is repeated until each wave velocity is analyzed.

**Step 4:** Repeat steps 2 and 3 until each frequency is analyzed, and a 2D dispersion image of the two receivers is obtained.

**Step 5:** The peak of the high amplitude distribution of the 2D dispersion image is regarded as the dispersion curve of the surface wave.

The time series data from a pair of receivers, \( s_{1,t} \) and \( s_{2,t} \), were obtained by surface wave seismic testing. They were transformed into the time–frequency data \( w_{1,t,f} \) and \( w_{2,t,f} \), respectively. Their temporal plots for a specific frequency are displayed together, as shown in **Fig. 2**. The first source-to-receiver offset is
Several numerical simulations of the surface wave seismic test were performed using the finite difference FLAC code. The synthesized seismic records in a two-layer earth model were simulated. There were 22,500 elements in the model. The numerical models were 30 m (horizontally) \times 30 m (vertically) with an element size of 0.2 m \times 0.2 m. The duration of the seismic records was set to 1 s with a time step of 0.2 ms. If the maximum frequency $f_{\text{max}} = 100$ Hz was considered, the minimum wavelength $\lambda_{\text{min}} = 2$ m would be obtained because the shear wave velocity of the upper-layer soil was assumed to be 200 m/s in this study. Therefore, the element size would be equal to $1/10 \lambda_{\text{min}}$. This implies a suitable element size for wave propagation analysis because it conforms to the criterion [19].

An axisymmetric numerical model was employed, and the boundary on the left side of the model is a symmetrical axis. A vertical impulsive load was applied to the intersecting point of the symmetrical axis and ground surface (Fig. 3). The impulsive load was assumed as a half sine-square function with a duration of 5 ms. The boundaries on the bottom and right side of the model were assumed as quiet (viscous) boundaries in the normal and shear directions to absorb most of the energy in the wave reflected from the boundaries. Twenty-nine receivers were placed on the ground surface in a linear array with equidistant spacing of 1 m in the following synthetic examples. The vertical particle motion velocity time histories at the receiver locations were recorded simultaneously. The velocity time histories were regarded as geophone-measured data in surface wave seismic testing.

The soils were assumed to be linear elastic materials, and material damping was neglected. The material parameters of the upper-layer soil were $\rho = 1800$ kg/m$^3$, $v_p = 346.4$ m/s, and $v_s = 200$ m/s, where $\rho$, $v_p$, and $v_s$ are the mass density, pressure wave velocity, and shear wave velocity, respectively. The bottom-layer soil was a half-space, and its material parameters were $\rho = 2100$ kg/m$^3$, $v_p = 866$ m/s, and $v_s = 500$ m/s.

To ensure that the dispersion image was contributed by the surface wave, and to avoid body wave effects, a minimum first source-to-receiver offset of 5 m was applied. If the frequency range of interest was 10 to 80 Hz and the shear wave velocity of the soil was 200 m/s, the corresponding shear wavelength would be from 2.5 to 20 m, which nearly satisfies the filtering criterion [20].

**MODEL A: TWO-LAYERED HORIZONTAL SOIL MODEL**

A softer soil layer with a thickness of 5 m that is overlaid on a homogeneous half-space in Model A is used to indicate the applicability of the CWT method when applied onto a horizontally layered model. The simulated model of the surface wave test is shown in Fig. 4. Fig. 5 shows that the dispersion images of the first source-to-receiver offsets is 10 m with four different spacing of receivers ($d = 1$, 2, 3, and 4 m) to evaluate the influence of the receiver spacing on the dispersion image. The theoretical dispersion curve for upper-layer soil thickness of 5 m denoted by a blue dashed line with circles is shown in the figures for
comparison purposes. From Fig. 5, it can be seen that the estimates of wave velocity obtained by using the method coincide with those of the theoretical values. For small receiver spacing, wider distribution of high-amplitude band in low-frequency range is shown in Fig. 5. Fig. 5d shows that a narrower distribution of high-amplitude band is obtained by considering the receiver spacing of 4 m. This implies that a good resolution of surface wave velocity is obtained by raising the spacing of receivers, which may be the reason for the long travel time of waves between receivers when a large receiver spacing provides a high-accuracy surface wave velocity.

MODEL B: TWO-LAYER MODEL WITH A VERTICAL FAULT

The CWT method was used in a vertical fault earth model to show its applicability regarding the lateral variation in layer thickness. The two-layer soils were separated by a single step-shaped line. The distance between the vertical fault and source was 15 m. The thickness of the upper-layer soil on the left side of the vertical fault was 5 m, and on the right side was 3 m (Fig. 6). To simplify the influence of lateral heterogeneity, the pair of receivers were placed on the same side of the vertical fault (i.e., both receivers were placed on the surface of a medium with the same thickness of the upper-layer soil).

For comparison purposes, in Fig. 7, the theoretical dispersion curves for upper-layer soil thickness are given as 5 and 3 m, respectively, denoted by a blue dashed line with circles and green dashed line with triangles. The theoretical
Dispersion curves were calculated by assuming the medium to be a horizontal layered model.

Fig. 7a shows the dispersion image when both receivers are located on the left side of the vertical fault. The estimated dispersion curve was picked from the dark dots (peak values) in a high-amplitude band of dispersion image. The estimated dispersion curve approaches the theoretical curve of the horizontally layered model with 5-m upper-layer soil thickness. Fig. 7b reveals that the theoretical dispersion curve of the horizontally

FIG. 5 Dispersion images of a two-layer horizontal soil model: (a) the first source-to-receiver offset = 10 m and receiver spacing = 1 m, (b) the first source-to-receiver offset = 10 m and receiver spacing = 2 m, (c) the first source-to-receiver offset = 10 m and receiver spacing = 3 m, and (d) the first source-to-receiver offset = 10 m and receiver spacing = 4 m.

FIG. 6 A two-layer model with a vertical fault for the surface wave seismic testing with the predicted soil layer by CWT (the orange and red colors are used to show the upper and lower soil layer, respectively).

FIG. 7 Dispersion images of a two-layer model with a vertical fault: (a) the first source-to-receiver offset = 10 m and receiver spacing = 4 m, and (b) the first source-to-receiver offset = 20 m and receiver spacing = 4 m.
layered model with the 3-m upper-layer soil thickness closely corresponds to the estimated dispersion curve obtained from the dispersion image. It can be observed that the dispersion image is dependent on the thickness of the upper-layer soil at the location of the pair of receivers.

**MODEL C: TWO-LAYER MODEL WITH AN INCLINING INTERFACE**

To verify the applicability of the CWT method on a high-lateral heterogeneity medium, a two-layer model with an inclining interface was examined. The thickness of the upper-layer soil varies linearly from 5 m (on the left side) to 3 m (on the right side), as shown in Fig. 8. Because the thickness of the upper-layer soil varied depending on the locations of receivers, the model had a high lateral heterogeneity. In this model, the spacing of the receivers was fixed at 4 m, and there were three distances (5, 15, and 25 m) applied between the first receiver and source. Fig. 9 shows the high-amplitude distribution shifts along the frequency axis with the varying receiver locations. The dispersion image is displayed in Fig. 9a, which shows that the estimated dispersion curve, indicated by the peak of the high-amplitude distribution, approaches the theoretical curve of the 5-m upper-layer soil. The dispersion curve represents the soil stratum on the left side of the model. The dispersion image in Fig. 9c shows that the estimated dispersion curve approaches the theoretical curve of the 3-m upper-layer soil because the two receivers were close to the right boundary of the model. Fig. 9b shows the dispersion image of the first source-to-receiver offset of 15 m. At this location, the thickness of the upper-layer soil is approximately 4 m. As expected, the estimated dispersion curve is between the theoretical curves for the thicknesses of the upper-layer soil of 3 and 5 m. Therefore, this method can detect the lateral variation of a shear wave velocity profile in a lateral heterogeneous medium.

**An Experimental Example**

The study area is situated on the campus of the Chaoyang University of Technology, Taichung, Taiwan. An earthfill of a thickness of approximately 8.1 m detected from a drilled borehole is overlaid on dark gray weathered sandstone. The earthfill is a layer of gray sandy silt with minute quantities of gravel, and it can be classified to be sandy silt (SM) according to the United Soils Classification System. The engineering properties of the earthfill from the experimental work are as follows: unit weight \( \gamma = 19.5 \text{kN/m}^3 \), natural water content \( w_n = 16 \% \), and friction angle \( \phi = 25^\circ \).

The impulsive source was derived by using a weighing 10 kg sledgehammer to beat a 0.2 m \( \times \) 0.2 m metal plate with the first source-to-receiver offset of 5 m and a receiver spacing of 4 m.
The recording time was only 1 s, with a sampling rate of 5000 samples per second. The dispersion image extracted using the CWT method is shown in Fig. 10. The theoretical dispersion curve for upper-layer soil thickness of 8 m denoted by a white dashed line with circles is shown in the figure. Fig. 10 shows that the asymptote at high frequencies approaches the wave velocities of the earthfill (190 m/s). From Fig. 10, it can be observed that the depth of the weathered bedrock estimated from the dispersion curve picked by the dark dots in dispersion image is approximately 8 m, which coincide with that of the neighboring borehole data, as shown in Fig. 11.

Conclusions

MASW is one kind of surface seismic test methods and from its result we can depict dispersion image and decide dispersion curve. However, it needs 12 or more receivers to determine the phase velocity for statistical redundancy. Therefore, it is still attractive when a seismic testing is only two receivers required.

In this study, the Morlet wavelet was selected in CWT to perform time–frequency analysis of seismic records from the two receivers; then, the slant stack procedures was performed to extract the dispersion image and to determine the dispersion curve for soil layers. These signal-processing procedures of the method proposed can be programmed. Therefore, the method has the advantages of using only two receivers and no need to make empirical judgment for phase unwrapping. The three numerical examples and an experimental example of seismic test were examined to evaluate the applicability of the proposed method for soil layers with lateral heterogeneity. The results show that the estimates of the wave velocities and the values from theoretical dispersion curve are in excellent agreement. This suggests that the method is applicable for extracting dispersion images and determining dispersion curves in surface wave seismic testing. A narrower distribution of high-amplitude band is obtained by raising spacing of receivers. The dispersion image is dependent on the thickness of the upper-layer soil at the center location of the pair of receivers in a lateral heterogeneous medium. Therefore, this method can detect the lateral variation of a shear wave velocity profile in a lateral heterogeneous medium. To conclude, we present that the method allows us to pick a dispersion curve without empirical judgment and using the seismic data from only two receivers to obtain an accurate 1D distribution of shear wave velocity versus depth.

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