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DAMPING CAPACITY OF BULK WHEAT

B. O. Hardin, K. O. Hardin, Z. Feng, I. J. Ross

ABSTRACT. *Results of resonant column tests on 23 specimens of bulk wheat, showing effects of strain amplitude, confining stress, wheat type (hard and soft red winter wheat), wheat density, and moisture content on wheat damping capacity are presented, along with models relating damping capacity to shear strain amplitude and to corresponding shear stiffness. Damping capacity is a primary property of bulk wheat needed to compute the seismic response of metal bins filled with wheat. Test results are presented to validate the use of staged testing procedures. Staged testing is desirable because it allows measurement of damping versus strain and damping versus shear modulus relationships for several different stress levels using a single specimen.*

Keywords. *Wheat, Damping, Shear stress, Strain, Vibration, Seismic analysis.*

All materials dissipate energy when deformed. Material damping capacity is a measure of the capability of a material to dissipate energy. The term damping is applied to energy dissipation during cyclic deformation or vibration. There are two sources of energy dissipation in the particulate structure of materials such as bulk wheat: (1) damping within individual grains, and (2) damping at contacts between grains. Although damping capacity is a primary property of bulk wheat needed to compute the seismic response of metal bins filled with wheat (Hardin et al., 1996), examination of the literature indicates that damping in bulk wheat has not been measured prior to the study reported herein.

The damping capacity of bulk wheat can be measured by torsional resonant column testing as described by the American Society of Testing and Materials (ASTM, 1998). For the resonant column test, a cylindrical specimen of bulk wheat is encased in a rubber membrane tube, attached to platens at each end of the cylinder. For the tests reported herein (Hardin, 1987), one end of the cylindrical specimen was fixed to prevent motion, while a sinusoidally varying torque was applied to the opposite end. The frequency of the applied torque was changed to produce first-mode resonance of the specimen. The measured resonant frequency and amplitude of applied torque were used to compute the shear modulus and damping capacity of bulk wheat in the specimen.

The article will present test results which show effects of strain amplitude, confining stress, wheat type (hard and soft red winter wheat), wheat density, and moisture content on the damping capacity of bulk wheat. Models relating damping capacity to shear strain amplitude and to corresponding shear stiffness will be presented.

CYCLIC SHEAR STRESS-STRAIN RELATION FOR PARTICULATE MATERIALS

The stress-strain relation for constant amplitude cyclic loading of a particulate material such as wheat is a hysteresis loop (fig. 1a). The slope of the loop reflects material stiffness (secant shear modulus G), and the area circumscribed by the loop is a measure of the energy dissipated during a cycle of loading. The ratio of energy dissipated in one loading cycle (A_L) to elastic strain energy stored at maximum strain (A_T) defines the hysteretic damping ratio:

$$D = \frac{1}{4\pi}\frac{A_L}{A_T} \qquad (1)$$

Figure 1b is a schematic representation of hysteresis loops for three different strain amplitudes (γ), where the apex of each loop is located on the dashed backbone curve (points a, b, and c). The effect of increasing strain amplitude γ is to decrease G and increase D. The reduction of G with an increase in γ is specified by defining the backbone curve in terms of G_{max} and τ_{max} (fig. 1c); G_{max} is the initial tangent modulus for the backbone curve (G approaches G_{max} as γ approaches 0), and τ_{max} is the simple shear strength of the particulate material. The reference strain:

$$\gamma_r = \frac{\tau_{max}}{G_{max}} \qquad (2)$$

can be used to normalize the backbone curve and define a modulus reduction function:

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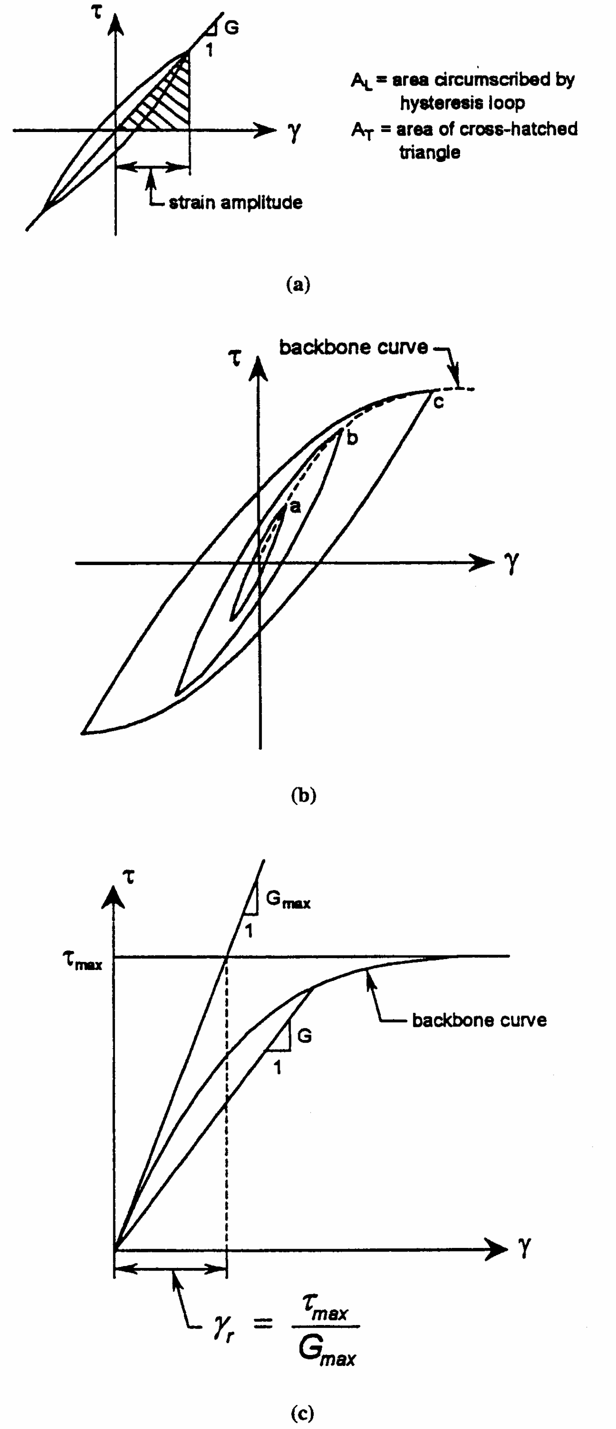


Figure 1–Effect of strain amplitude on cyclic stress-strain relation.

$$\frac{G}{G_{max}} = f_1\left(\frac{\gamma}{\gamma_r}\right) \qquad (3a)$$

that is nearly independent of state of stress in the particulate material (Hardin and Drnevich, 1972).

Test results presented herein will show that a second function, f_2, can be defined to model the relationship between D and G:

$$\frac{D - D_{min}}{D_{max} - D_{min}} = f_2\left(\frac{G}{G_{max}}\right) \qquad (3b)$$

where D_{max} is the damping ratio at large strain amplitude (maximum value of D), D_{min} is the damping ratio for strain amplitudes approaching zero (minimum value of D), and the ratio $(D - D_{min})/(D_{max} - D_{min})$ is called relative damping. Combining equations 3a and 3b gives a third function relating D and γ. Test results for bulk wheat will be used to define the function:

$$\frac{D - D_{min}}{D_{max} - D_{min}} = f_3\left(\frac{\gamma}{\gamma_r}\right) \qquad (3c)$$

DAMPING MODELS

Two models used to approximate damping behavior of bulk wheat are shown schematically in figure 2. The D versus γ model (fig. 2a) approximates relationships between damping ratio and shear strain amplitude for different stress levels (σ'_o). Damping ratio exhibits its

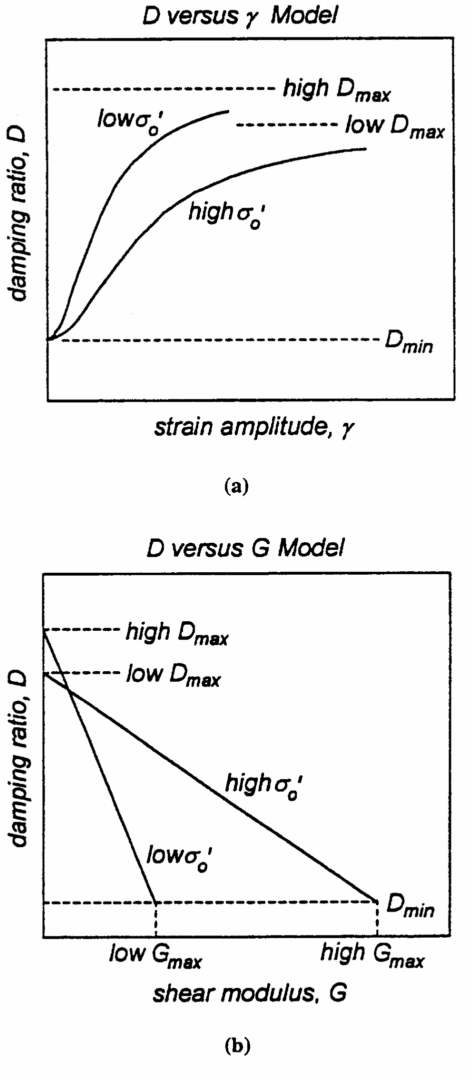


Figure 2–Damping models for bulk wheat: (a) D versus γ model, (b) D versus G model.

minimum value (D_{min}) at small strain amplitudes. The value of D increases with strain amplitude, approaching D_{max} asymptotically at very large strain. The rate of increase of D with γ and value of D_{max} depend on the stress level (σ'_0). The D versus G model (fig. 2b) approximates relationships between damping ratio and shear modulus for different initial isotropic states of stress (σ'_0). As shear strain amplitude increases (with constant σ'_0), D increases from D_{min} to D_{max} while G decreases from G_{max} to zero. The value of D_{max} is higher for lower values of σ'_0. At small strain amplitudes, where G equals G_{max}, the damping ratio has its minimum value (D_{min}).

Test results indicate that D_{min} increases with increasing wheat moisture content, but is independent of confinement σ'_0. Evidently the primary source of damping at small strains is deformation of individual grains, since damping at contacts between grains is expected to vary with confinement (Duffy and Mindlin, 1957). The damping capacity of individual grains may be expected to increase with moisture content, as increasing moisture causes grains to soften. Examination of the literature indicates that energy dissipation in individual grains has not been measured.

TESTING PROGRAM

Specimens of soft and hard red winter wheat were tested in loose and dense states (table 1). Specimens of wheat for each of the four combinations of hardness and density were tested at moisture contents equal to 8%, 13%, and 18% (w.b.). Samples of wheat in quantities sufficient for test purposes were placed on a large perforated tray in an environmental control chamber. The temperature in the convection chamber was maintained at 25°C and the relative humidity varied to obtain the desired grain moisture content (± 0.25%, w.b.). Moisture content was measured by the oven method (*ASAE Standards*, 1995). Conditioned grain samples were sealed in glass containers and maintained at room temperature.

Table 1. Testing program

% (wb)	Test No.	Void Ratio	σ'_0-values
Soft Red Winter Wheat (loose)			
8	15	0.74	17.2, 34, 69, 138, 276
13	1	0.71	17.2, 34, 69, 138, 276
	2	0.70	34, 69, 138, 276
	3	0.69	69, 138, 276
	10	0.69	138, 276
	23	0.64	276
	22	0.71	34, 69, 138, 276
18	17	0.72	17.2, 34, 69, 138, 276
Soft Red Winter Wheat (dense)			
8	16	0.62	17.2, 34, 69, 138, 276
13	4	0.62	17.2, 34, 69, 138
	5	0.57	34, 69, 138, 276
	6	0.57	69, 138, 276
	7	0.57	138, 276
	8	0.55	276
18	18	0.59	17.2, 34, 69, 138, 276
Hard Red Winter Wheat (loose)			
8	13	0.73	17.2, 34, 69, 138, 276
13	11	0.71	17.2, 34, 69, 138
	24	0.64	276
18	19	0.78	17.2, 34, 69, 138, 276
Hard Red Winter Wheat (dense)			
8	14	0.61	17.2, 34, 69, 138, 276
13	12	0.61	17.2, 34, 69, 138, 276
18	20	0.65	17.2, 34, 69
	21	0.58	138, 276

The range of void ratios (e) for tests in the loose group was 0.64 to 0.78, and for the dense group 0.55 to 0.65 with an overlap at 0.64 to 0.65. Most specimens were tested at five isotropic confining stress levels, σ'_0 equal to 17.2, 34, 69, 138, and 276 kPa, in five stages as described in the next paragraph. The prime is used with σ_0 to indicate that particulate material behavior depends on effective stress. The voids or pores of test specimens were filled with air at atmospheric pressure. Thus total stress σ_0 is equal to effective stress σ'_0.

Each five-stage test started with σ'_0 equal to 17.2 kPa and low amplitude of torsional vibration (small shear strain amplitude). Electrical current through the coils of the resonant column apparatus was increased in steps, with resonant frequency and driving current measured at each current level. Shear modulus and damping ratio were computed from these measured quantities. Measurements were made at 10 to 15 amplitudes of vibration, until maximum current capacity of the apparatus was reached. Thus, the variation of G and D with γ were determined for σ'_0 equal to 17.2 kPa (stage 1).

Between stages, power was reduced to produce very small vibration (small shear strain). To begin stage 2, the chamber pressure was increased to σ'_0 equal to 34 kPa; then, vibration amplitude was increased in steps and stage 2 measurements were made. The same procedure continued for stages 3, 4, and 5, with σ'_0 equal to 69, 138, and 276 kPa, respectively. To assess effects of staged testing on shear modulus and damping measurements, six loose specimens and five dense specimens of soft red winter wheat at 13% (w.b.) were tested with different numbers of stages (1 to 5). The number of stages for a given specimen is equal to the number of σ'_0-values shown in table 1 for that specimen.

Values of G and D were measured by resonant column testing where measurements are made at the resonant frequency of the testing system (apparatus plus specimen). Resonant frequencies for all measurements reported herein were in the range 20 to 130 Hz. Void ratios were computed from measurements of specimen volume, weight of wheat, and specific gravity of wheat grains.

DAMPING RATIO VERSUS SHEAR STRAIN AMPLITUDE AND EFFICACY OF REFERENCE STRAIN

The effect of σ'_0 on the relationship between damping ratio and shear strain amplitude is illustrated by results from tests 12, 16, and 18 plotted in figure 3 (left-hand side). The rate of increase of D with γ and corresponding value of D_{max} are larger for lower values of σ'_0. However, the curves for different values of σ'_0 approach a common value of D_{min} at γ equal to 0. The data clearly show that D_{min} increases with increasing specimen moisture content.

Values of damping ratio measured for a given specimen subjected to five levels of confinement σ'_0 are plotted versus normalized strain (γ/γ_r) on the right-hand side of

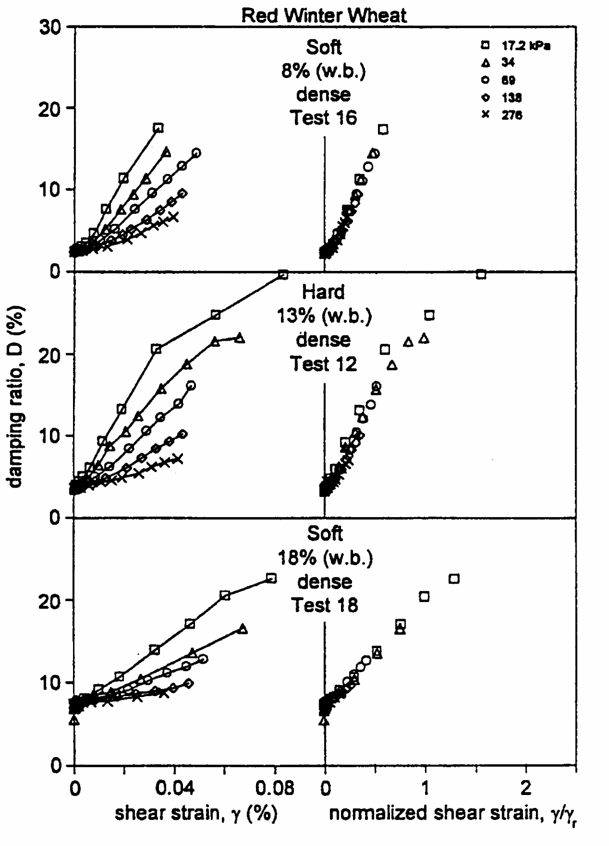


Figure 3–Effect of confining stress on D versus γ and D versus γ/γ_r relationships.

figure 3. These data show that use of reference strain (γ_r) to normalize shear strain leads to a relationship between D and γ/γ_r for a given specimen that is approximately independent of σ'_0.

DETERMINATION OF REFERENCE STRAIN

Reference strain is defined by the ratio of simple shear strength (τ_{max}) to maximum shear modulus (G_{max}) (see figure 1c and eq. 2). Values of G_{max} measured for each level of confinement for each of the tests listed in table 1 are plotted versus σ'_0 in figure 4. Corresponding values of τ_{max} were computed using the strength model for bulk wheat presented by Hardin et al., (1990). The algorithm for computation of simple shear strength by this model is given in figure 5. The strength model coefficients $\phi_{\mu o\sigma}$ and $d_{o\sigma}$ required in figure 5 are given in table 2. These coefficients were determined by Hardin et al. (1990) from triaxial compression tests on the same bulk wheat materials tested herein. Computed values of τ_{max} are listed in table 3 for each of the tests in table 1. Finally, values of γ_r equal to τ_{max}/G_{max} used to normalize strain herein are plotted versus σ'_0 in figure 6.

Parenthetically, it should be noted that the R_{max} equation (next to last equation in fig. 5) is based on equation 14 in the original strength model reference (Hardin, 1985). An error in the sign of one of the R_{cv} terms was made in copying the R_{max} equation to equation 5a in Hardin et al. (1990) and to equation 7a in Hardin et al. (1996).

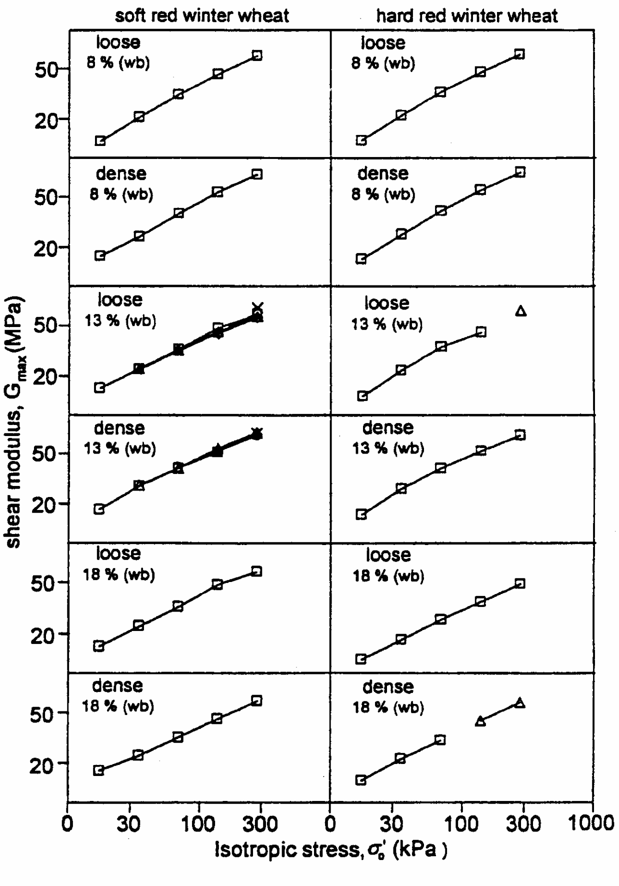
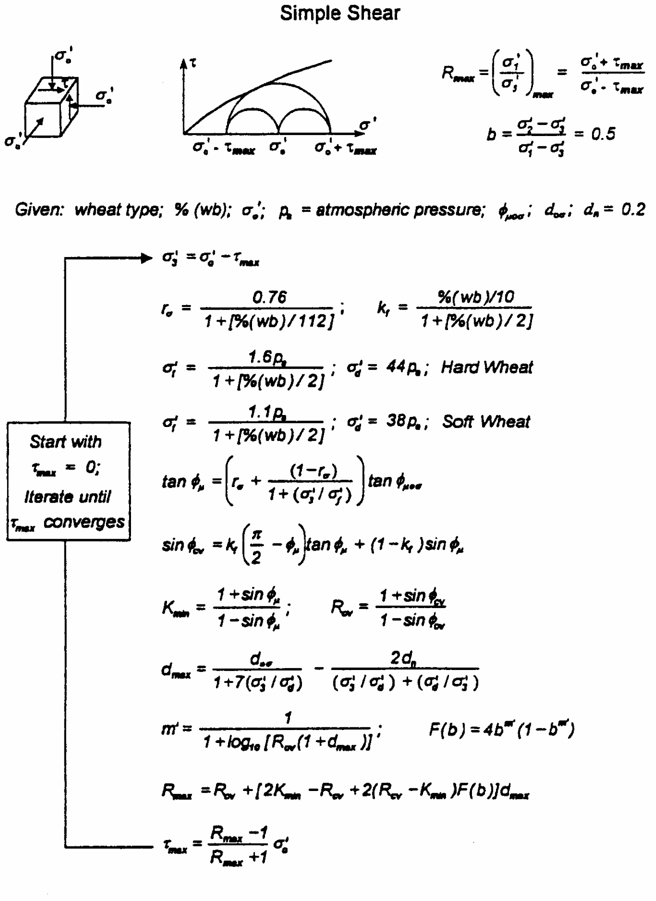


Figure 4–G_{max} values used to compute reference strain.



$$R_{max} = \left(\frac{\sigma'_1}{\sigma'_3}\right)_{max} = \frac{\sigma'_o + \tau_{max}}{\sigma'_o - \tau_{max}}$$

$$b = \frac{\sigma'_2 - \sigma'_3}{\sigma'_1 - \sigma'_3} = 0.5$$

Given: wheat type; % (wb); σ'_o; p_a = atmospheric pressure; $\phi_{\mu o\sigma}$; $d_{o\sigma}$; $d_n = 0.2$

$$\sigma'_3 = \sigma'_o - \tau_{max}$$

$$r_\sigma = \frac{0.76}{1 + [\%(wb)/112]}; \qquad k_f = \frac{\%(wb)/10}{1 + [\%(wb)/2]}$$

$$\sigma'_f = \frac{1.6 p_a}{1 + [\%(wb)/2]}; \; \sigma'_d = 44 p_a; \; \text{Hard Wheat}$$

$$\sigma'_f = \frac{1.1 p_a}{1 + [\%(wb)/2]}; \; \sigma'_d = 38 p_a; \; \text{Soft Wheat}$$

$$\tan\phi_\mu = \left(r_\sigma + \frac{(1 - r_\sigma)}{1 + (\sigma'_3/\sigma'_f)}\right)\tan\phi_{\mu o\sigma}$$

$$\sin\phi_{cv} = k_f\left(\frac{\pi}{2} - \phi_\mu\right)\tan\phi_\mu + (1 - k_f)\sin\phi_\mu$$

$$K_{min} = \frac{1 + \sin\phi_\mu}{1 - \sin\phi_\mu}; \qquad R_{cv} = \frac{1 + \sin\phi_{cv}}{1 - \sin\phi_{cv}}$$

$$d_{max} = \frac{d_{o\sigma}}{1 + 7(\sigma'_3/\sigma'_d)} - \frac{2 d_n}{(\sigma'_3/\sigma'_d) + (\sigma'_d/\sigma'_3)}$$

$$m' = \frac{1}{1 + \log_{10}[R_{cv}(1 + d_{max})]}; \qquad F(b) = 4b^{m'}(1 - b^{m'})$$

$$R_{max} = R_{cv} + [2K_{min} - R_{cv} + 2(R_{cv} - K_{min})F(b)]d_{max}$$

$$\tau_{max} = \frac{R_{max} - 1}{R_{max} + 1}\sigma'_o$$

Figure 5–Algorithm for computing τ_{max}.

Table 2. Strength model coefficients

Moisture Content % (w.b.)	Loose $\phi_{\mu o\sigma}$	Loose $d_{o\sigma}$	Dense $\phi_{\mu o\sigma}$	Dense $d_{o\sigma}$
Soft Red Winter Wheat				
8	26.3	0.150	25.2	0.513
13	26.3	0.150	27.1	0.412
18	32.0	0.100	32.5	0.400
Hard Red Winter Wheat				
8	24.4	0.071	22.7	0.400
13	23.1	0.150	23.1	0.445
18	27.0	0.150	28.6	0.400

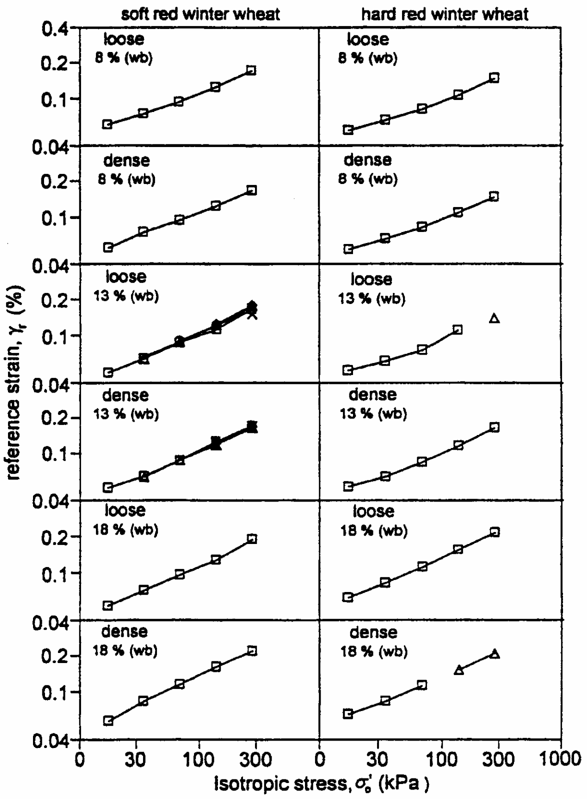


Figure 6–Values of reference strain used to normalize measured shear strains.

DAMPING RATIO VERSUS SHEAR STRAIN AMPLITUDE MODEL

Relative damping ratio values $[(D - D_{min})/(D_{max} - D_{min})]$ for dense and loose specimens of red winter wheat are plotted versus normalized shear strain (γ/γ_r) in figures 7 and 8, respectively. Values for five stress levels (σ'_o) are approximated by single curves in these normalized plots. The curves approximating measured data are defined by the D versus γ model:

$$\frac{D - D_{min}}{D_{max} - D_{min}} = \frac{a\left(\frac{\gamma}{\gamma_r}\right)^n}{1 + a\left(\frac{\gamma}{\gamma_r}\right)^n} \tag{4a}$$

Table 3. Values of shear stress, τ_{max}

σ'_3 (kPa)	8% (wb) Loose	8% (wb) Dense	13% (wb) Loose	13% (wb) Dense	18% (wb) Loose	18% (wb) Dense
Soft Red Winter Wheat τ_{max} (kPa)						
17.2	8.3	9.8	8.1	9.6	9.0	10.6
34.5	16.0	19.1	15.5	18.6	17.1	20.4
69.0	30.6	36.8	29.4	35.7	32.5	39.1
137.9	58.2	70.4	56.0	68.2	62.1	75.0
275.9	110.8	133.7	106.9	129.9	119.6	143.9
Hard Red Winter Wheat τ_{max} (kPa)						
17.2	7.5	8.8	7.5	9.0	8.3	9.9
34.5	14.4	17.3	14.3	17.5	15.8	19.2
69.0	27.4	33.2	27.0	33.5	29.8	36.6
137.9	51.9	63.2	51.2	63.7	56.5	69.7
275.9	98.5	119.4	97.0	120.5	107.7	132.9

where n is equal to 1.6, and values of a and D_{min} are given in tables 4 and 5, respectively. Values of D_{max} are defined by:

$$D_{max} = D_{min} + \frac{D_o - D_{min}}{1 + A\left(\frac{\sigma'_o}{p_a}\right)} \tag{4b}$$

in which p_a is atmospheric pressure, A equals 0.06, and D_o equals 0.36. The maximum value of D_{max} is 36% and occurs for σ'_o equal to 0. Values of standard error of

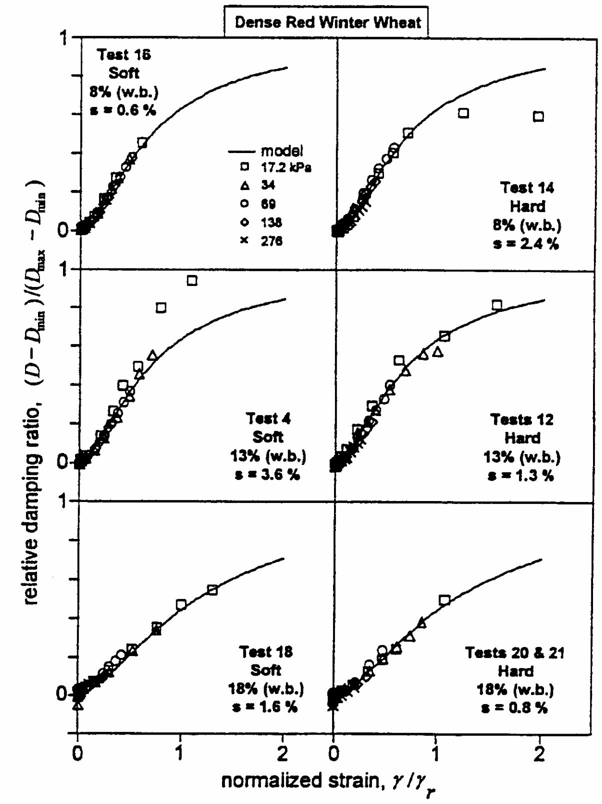


Figure 7–Relative damping versus normalized strain for dense red winter wheat.

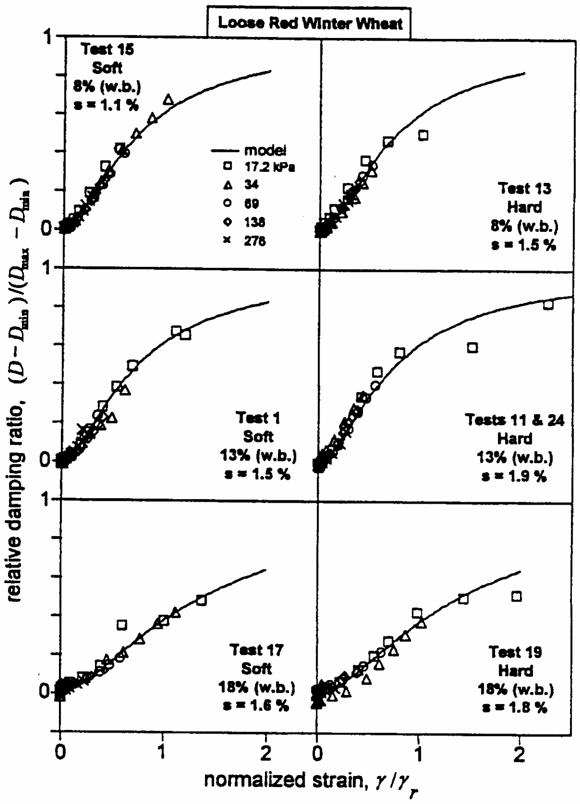


Figure 8–Relative damping versus normalized strain for loose red winter wheat.

Table 4. Value of *a* for red winter wheat

% (w.b.)	*a* Loose	*a* Dense
8	1.6	1.8
13	1.6	1.8
18	0.6	0.8

Table 5. Value of D_{min} for red winter wheat

% (w.b.)	D_{min} (%) Soft	D_{min} (%) Hard
8	2.2	2.2
13	3.8	3.8
18	6.9	6.4

estimates (s) are shown in figures 7 and 8 comparing the D versus γ model to each of the 12 sets of data for soft and hard, dense and loose, red winter wheat at 8, 13, and 18% (w.b.). In 10 of the 12 cases, s is less than 2% damping.

DAMPING RATIO VERSUS SHEAR MODULUS MODEL

Damping ratio values (D) for dense and loose specimens of red winter wheat are plotted versus shear modulus (G) in figures 9 and 10, respectively. Values for five stress levels (σ'_0) are shown. The straight lines approximating measured data are defined by the D versus G model:

$$\frac{D - D_{min}}{D_{max} - D_{min}} = 1 - \frac{G}{G_{max}} \qquad (5)$$

where D_{max} is defined by equation 4b and values of D_{min} are given in table 5. Values of standard error of estimates (s) are shown in figures 9 and 10 comparing the D versus G

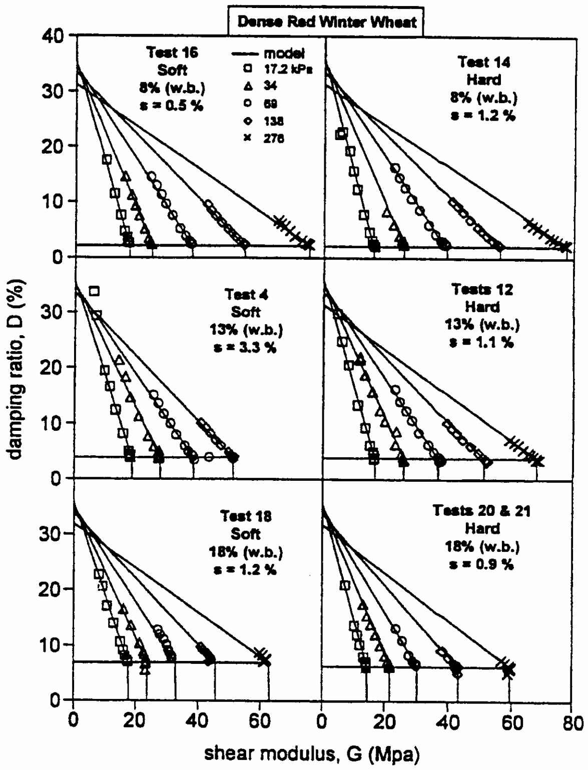


Figure 9–Damping ratio versus shear modulus for dense red winter wheat.

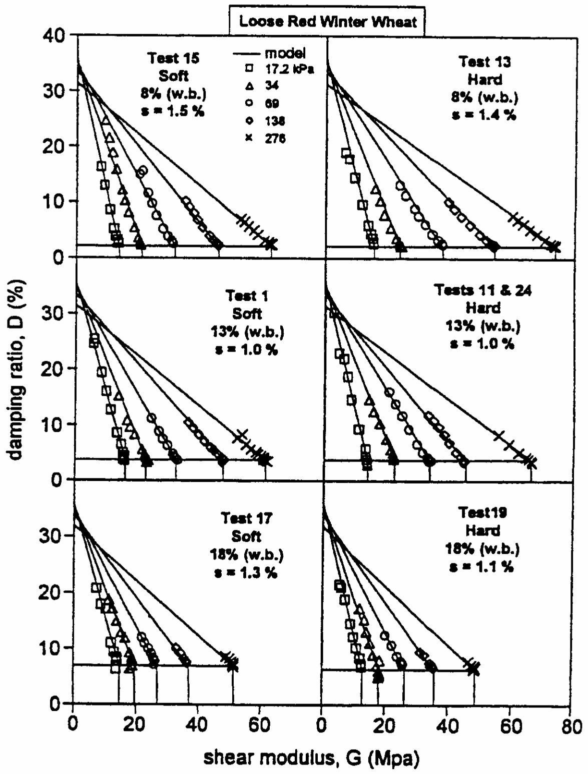


Figure 10–Damping ratio versus shear modulus for loose red winter wheat.

model to each of the 12 sets of data. For 11 of 12 data sets, s is less than or equal to 1.5% damping.

VALIDATION OF STAGED TESTING

Staged testing is desirable because it allows measurement of D versus γ and D versus G relationships for several different stress levels (σ'_0) using a single specimen. To assess the validity of this procedure, six loose specimens and five dense specimens of soft red winter wheat at 13% (w.b.) were tested with different numbers of stages (1 to 5).

Relationships between relative damping and normalized shear strain for dense and loose specimens of soft red winter wheat are shown in figures 11 and 12, respectively. The number of stages for a given specimen is equal to the number of σ'_0-values shown in table 1 for that specimen. For example, test 4 includes four stages, σ'_0 equal to 17.2, 34, 69, and 138 kPa; test 5 includes four stages, σ'_0 equal to 34, 69, 138, and 276 kPa; while test 8 includes only one stage σ'_0 equal to 276 kPa. The curves in figures 11 and 12 are defined by the D versus γ model (eqs. 4 and tables 4 and 5) This is the same model used to approximate staged test data in figures 7 and 8. Comparison of data points to model curves in figures 11 and 12 indicates no evident effect of number of testing stages on measurement of the D versus γ relationship. For 10 of 11 data sets, s is less than or equal to 2% damping.

Relationships between damping and shear modulus for the tests involving different numbers of stages are given for dense and loose specimens of soft red winter wheat in figures 13 and 14, respectively. The straight lines in figures 13 and 14 are defined by the D versus G model (eqs. 4b and 5, table 5). This model was used previously to approximate staged test data in figures 9 and 10. Comparison of data points to model curves in figures 13 and 14 indicates no evident effect of number of testing stages on measurement of the D versus G relationship. For 10 of 11 data sets, s is less than or equal to 1.4% damping.

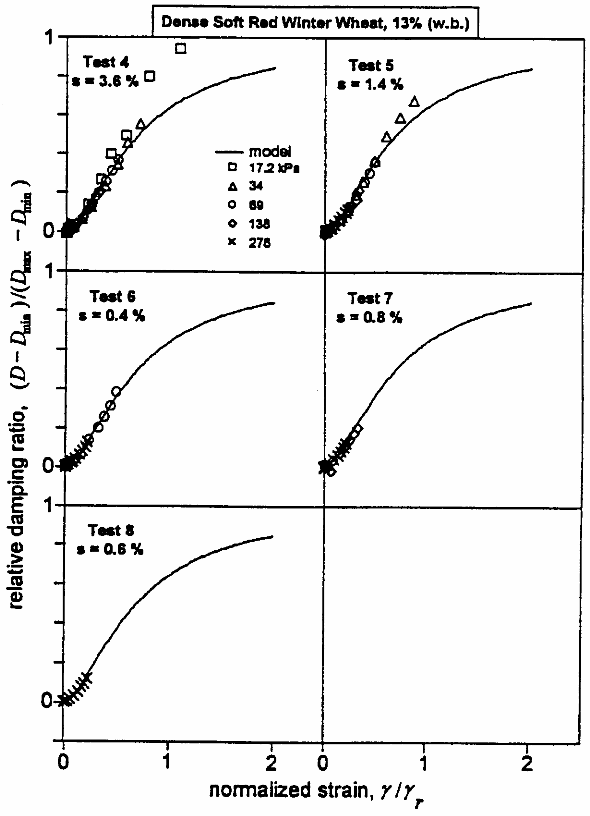


Figure 11–Relative damping versus normalized strain for different number of stages, dense soft red winter wheat.

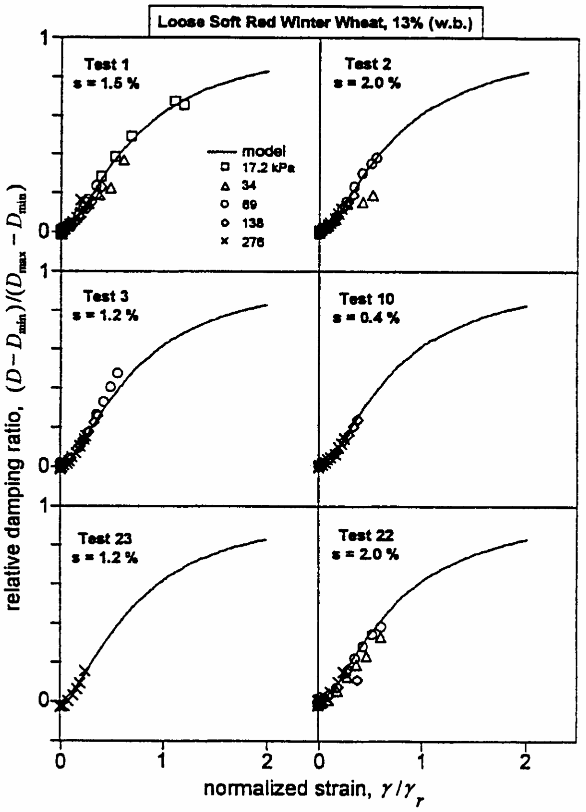


Figure 12–Relative damping versus normalized strain for different number of stages, loose soft red winter wheat.

SUMMARY AND CONCLUSIONS

Values of damping capacity measured by resonant column tests on 23 specimens of bulk wheat have been presented. Effects of strain amplitude, confining stress, wheat type (hard and soft red winter wheat), wheat density, and moisture content on damping capacity have been evaluated. Evidently, these are the first measurements of bulk wheat damping capacity to be reported in the literature.

Damping capacity increases with strain amplitude from a minimum value D_{min} at small strains to a maximum value D_{max} for very large strains. The value of D_{min} increases with increasing moisture content but is independent of confinement σ'_0. Evidently the primary source of damping at small strains is deformation of individual grains, since damping at contacts between grains is expected to vary with confinement. The value of D_{max} varies with σ'_0.

The value of D_{min} for soft red winter wheat increases with increasing moisture content from about 2.2% at 8% (w.b) to about 6.9% at 18% (w.b.). The corresponding D_{min} range for hard red winter wheat is about 2.2 to 6.4% (table 5). The value of D_{max} decreases with increasing σ'_0

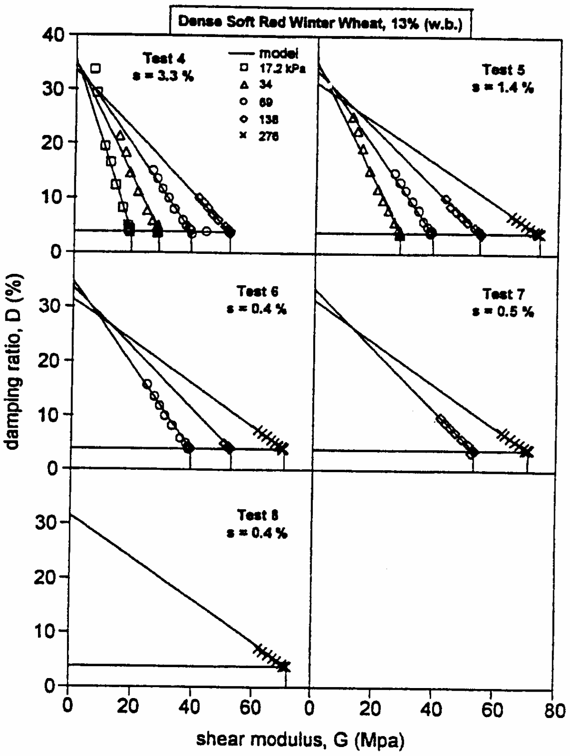


Figure 13–Damping ratio versus shear modulus for different number of stages, dense soft red winter wheat.

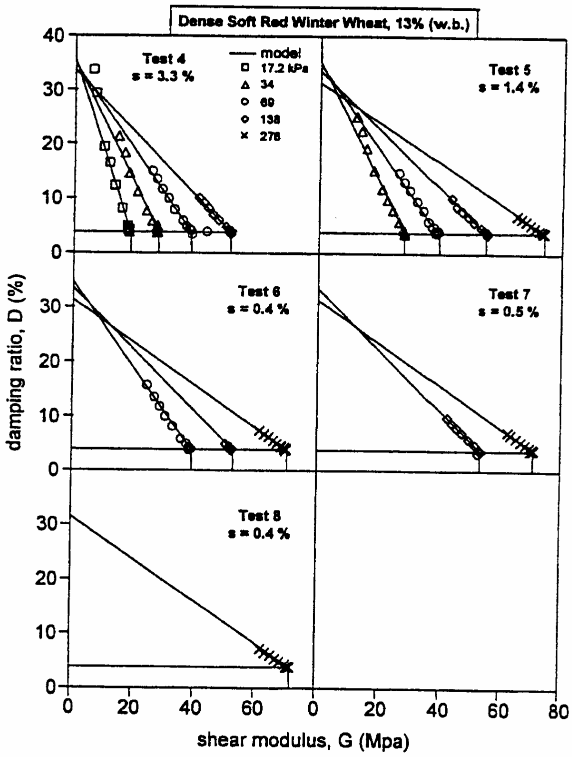


Figure 14–Damping ratio versus shear modulus for different number of stages, loose soft red winter wheat.

as shown by equation 4b which indicates that D_{max} has a maximum value of 36% at σ'_0 equal to 0.

Models relating damping capacity to shear strain amplitude (D vs γ model) and to shear stiffness (D vs G model) have been defined. The rate of increase of D with strain amplitude γ is larger for lower values of σ'_0. However, data are presented to show that use of reference strain γ_r which is equal to τ_{max}/G_{max} to normalize strain leads to a relationship between relative damping $[(D - D_{min})/(D_{max} - D_{min})]$ and normalized strain (γ/γ_r) that is independent of σ'_0 (D vs γ model). As strain amplitude increases, the value of D increases while the corresponding value of shear modulus G decreases from its maximum value G_{max}. The relationship between D and G is approximately linear. Data are presented to show that the relationship between relative damping and normalized shear modulus G/G_{max} is independent of σ'_0 (D vs G model).

Finally, test results are presented that indicate the validity of staged testing. Staged testing is desirable because it allows measurement of D versus γ and D versus G relationships for several different stress levels (σ'_0) using a single specimen.

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