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An Application of Constitutive Modeling in FLAC for Cohesionless Soils

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Abstract

This study was to implement a relative simple but realistic model for cohesionless soils into FLAC to perform analyses. The model is named "RD model" and can be applied to plane strain and axisymmetric problems. The model is based on the concept of critical state soil mechanics and is capable of simulating dilative and contractive behavior of granular soils. The RD model is developed using FISH language. The responses of a soil element subjected to an axial compression test were used to illustrate the behavior of the RD model. The soil element was loaded monotonically while maintaining the lateral stress constant. Two conditions of the sands were simulated, that is, initially loose and dense states. The terms "loose" and "dense" are related not only to the void ratio of the soil but also to the magnitude of the stress level. To show the application of the developed model, an example of a circular surface foundation was simulated. The stress-strain results of the RD model for the foundation were compared to that of the Mohr-Coulomb model under the same loading conditions. Therefore, three sets of soil parameters for the dense and loose states of the RD model, and for the Mohr-Coulomb model were used. The results of the stress versus displacement of the simulations are presented. It is assumed that similar critical state behavior exists in cyclic loading. An attempt was made to simulate cyclic behavior of cohesionless soil. The results of the simulation for element level are presented.

1. Introduction

Because soils have irreversible and non-linear behavior during loading, linear elastic model is unsuitable for most analyses except for small strain analysis, such as elastic wave analysis in soils and foundation vibration. Some simplified non-linear elastic models and "equivalent-linear" analysis methods had been widely used. However, they do not account for plastic yielding of the soil. Plasticity based models are superior because the non-linear constitutive relationship and permanent plastic strain can be accounted for properly. Furthermore, it may be possible to model the hysteretic behavior in the stress-strain relationship, and thereby the material damping when performing a dynamic analysis.

This study was to implement a relative simple but realistic plastic model for cohesionless soils into FLAC (FLAC, 1995) for plane strain and axisymmetric problems. The general responses of a soil element subjected to an axial compression test are presented. To show the application of the developed model, an example of a circular surface foundation was simulated. Three sets of soil parameters were used in the vertical loading simulations for the foundation. The results of the stress versus displacement of the simulations are presented.

A cyclic plastic model that can capture the dynamic behavior of cohesionless soils including dilation and contraction under shearing and is suitable to be incorporated into FLAC is currently not available. The challenge to develop such a model is formidable because there are many factors influencing the modulus and damping in soil, suc as strain amplitude, effective mean principal stress, void ratio, and number of loading cycles (Hardin and Drnevich, 1972). An attempt was made to simulate cyclic behavior of cohesionless soil.

2. Components of Plasticity Theory

The theory of plasticity may be the best tool for developing soil models that can be used for monotonic and dynamic loading. It is because that plasticity theory has a systematic way to describe the yielding and irreversible behavior of soils. Plasticity based models typically consist of a yield criteria, a plastic potential function (a flow rule) and a hardening rule. The following is an overview of the basis of plasticity theory.

2.1 Yield Criteria

Yield criteria defines stress states when the material starts to undergo plastic deformation. The yield criteria can be expressed by a combination of stress states. Establishment of the criteria is typically based on the results of laboratory tests with some simplifications to develop convenient functional relationships. The yield criteria can be written as:

$$f = 0 \qquad \text{Eq. 1}$$

in which f is the yield function. For stress states that $f \leq 0$, the material is behaving elastically. For $f = 0$, plastic strain is occurring.

2.2 Plastic Potential Function and Flow Rule

A plastic potential function g is used to define the relative magnitudes of plastic strains, e.g., the ratio of plastic volumetric strain to plastic shear strain. The plastic strain increment is normal to the plastic potential function. For example, when the plastic potential function is plotted in q-p' (deviator-effective mean stress) space, the plastic volumetric strain is aligned with p' and the plastic shear strain is aligned with q. The plastic strain is assumed to be a function of the stress state, not of the stress increment. The flow rule is related to the plastic potential function and it defines the ratio of the components of the plastic strain increments for a given load increment. We can express the flow rule as:

$$de_{ij}^{p} = dl \, \frac{\partial g}{\partial s_{ij}} \qquad \text{Eq. 2}$$

where de_{ij}^{p} = plastic strain tensor,

s_{ij} = stress tensor,

g = plastic potential function, and

$d\lambda$ = a factor of proportionality.

The factor, dλ, can be determined from the consistency condition. The consistency condition is used to ensure that the stress state remains on the yield surface (Chan and Han, 1988). The flow rule is said to be associated when the plastic potential function, *g*, is the same as the yield function, *f*. Otherwise, it is a non-associated flow rule. For sands, a non-associated flow rule is usually assumed to be consistent with the results of laboratory tests.

2.3 Hardening Rule

The yield surface may expand (strain hardening) or contract (strain softening materials). The hardening rule defines how the yield surface changes with accumulative plastic strain, for instance, it specifies the evolution of the subsequent yielding surface. Isotropic hardening, kinematic hardening and mixed hardening rules have been developed (Chen and Han, 1988).

3 The User-Defined Soil Model for FLAC

This section introduces a relative simple but realistic model for cohesionless soils for performing numerical simulations in FLAC. The model is named "RD model" after its initial developer, Dr. Richard Deschamps of Purdue University. It can be applied to plane strain and axisymmetric problems. The model is based on the concept of critical state soil mechanics and is capable of simulating dilative and contractive behavior of granular soils. The RD model is developed using FISH language. FISH is a special programming language of FLAC to conduct numerical analyses. In addition, it allows users to construct user-defined constitutive models. In FLAC, user-defined models can be developed in an incremental fashion over each stepping. Once the model is compiled successfully, it behaves as a built-in model except somewhat slower.

As described in Section 2, the yielding criterion, f, and the potential function, g, for a plastic model have to be defined. The yielding criterion and potential function used for the RD model are based on Drucker-Prager model. They are defined as following:

$$f(t, s) = t + q_f s \qquad \text{Eq. 3}$$

$$g(t, s) = t + q_y s \qquad \text{Eq. 4}$$

where $t = \sqrt{\frac{1}{2} s_{ij} s_{ij}}$,

$s = \frac{s_{kk}}{3}$,

q_f , q_y = material parameters,

s_{ij} = deviatoric-stress tensor, and

s_{kk} = normal-stress tensor.

The hardening rule established in the RD model is an isotropic hardening rule. It governs the expanding and contracting of the yielding surface and the surface of the plastic potential function. This is done by updating the material parameters q_f and q_y during plastic straining. In the RD model, the determination of "loose" or "dense" state of soil is similar to that of the critical state soil mechanics (Wood, 1990). The "state parameter" is the relative position with respect to the critical state line and governs the amount and rate of dilation and contraction. Once the state parameter is computed, the q_f and q_y are updated according to the state parameter of the soils for each element and for each step.

4. The Basic Aspects of the RD Model

In this section, the basic aspects of the stress-strain response of sand captured by the RD model are introduced (Leonards et al., 1995). The response of a soil element subjected to an axial compression test will be used to illustrate the behavior. The soil element is loaded monotonically while maintaining the lateral stress constant. Figure 1a shows the shear stress-shear strain response, and Figure 1b shows the stress path associated with the applied stresses. The two curves in Figure 1a represent sands that are initially loose and dense. The dense sand is stiffer and stronger than the loose sand. It also exhibits a peak shear stress followed by a softening in strength to the critical state condition at large strains, while for the loose sand the shear stress increases monotonically to the critical state value. For the same stress path, the critical state strength is the same at large strains, regardless of the initial void ratio of the sand. The unique critical state shear strength is explained in Figure 1c. Loose sands tend to contract upon shearing while dense sands tend to dilate. The loose and dense sands tend to reach the same critical state void ratio and shear strength at large strain. As illustrated in Figure 1d, the critical state void ratio depends on stress level and also on the stress path to which the soil is subjected. A point representing a void ratio and generalized normal stress that is above the critical state line will contract on shearing, while a point that is below the line will tend to dilate. Thus, the terms "loose" and "dense" are related not only to the void ratio but also to the stress level. The state parameter which is the relative position with respect to the critical state line, governs the amount and rate of dilatancy as well as the form of the stress-strain relationship.

5. Simulations Using the RD Model

To show the application of the RD model, an example of a circular surface foundation was simulated. The triaxial loading test on an element was simulated first using the parameters of the RD model for the initial dense and loose sands. The stress-strain results were compared to those of Mohr-Coulomb model under the same loading conditions. Therefore, three sets of soil parameters for the dense and loose states of the RD model, and for the Mohr-Coulomb model were used. Figure 2 shows the stress versus strain curves of the triaxial compression tests of the soil models.

The circular surface foundation (radius = 3.375 m) was assumed smooth at the base, and axisymmetric condition was assumed. A downward velocity was applied to the gridpoints of the foundation to simulate a loading. The RD model or the Mohr-Coulomb model was assigned to all the elements for each simulation. The mesh is shown in Figure 3. The parameters for the Mohr-Coulomb model are as following:

Shear modulus: 100 MPa
Bulk modulus: 200 MPa
Density: 2000 kg/m^3
Cohesion: 0 Mpa
Tensile strength: 0 MPa
Friction angle: 30 degree
Dilation angle: 0 degree

For the RD model, the critical state friction angle (ϕ_{cs}) was selected as 30 degree, which was used to calculate the yielding surface at its critical state. The soil parameters were given similar to the modified Cam-clay model to determine the "state parameter". The dilation and friction angles of the soils were modified during the simulations according to the variations of the "state parameter" and the plastic strains.

In addition to the axisymmetric calculations in FLAC, another simulation was made for the Mohr-Coulomb model using FLAC^{3D} (FLAC^{3D}, 1997) for comparison. The Mohr-Coulomb model parameters were the same as those used in the axisymmetric simulation. The soil body was simulated as a quarter cylinder, and the foundation was subjected to the same loading rate as in the axisymmetric configuration. All the stress versus displacement results of the simulations are shown in Figure 4. The response of the axisymmetric and the 3D cases using the Mohr-Coulomb model are very close. The response of the loose soil using the RD model is also converged to the same stress as that using the Mohr-Coulomb model. For the dense sand using the RD model, the stress is much higher and shows a sign of softening too.

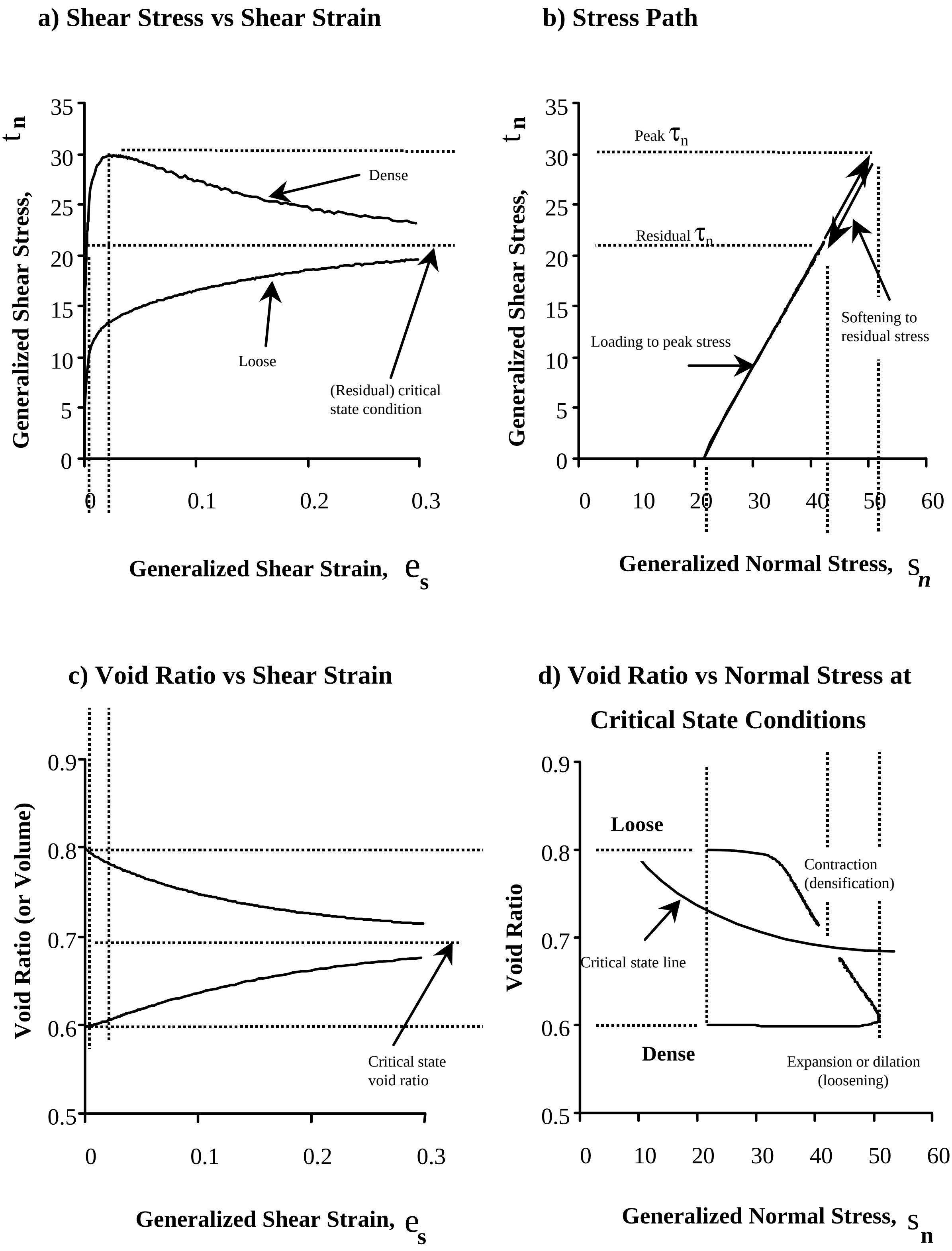

Figure 1 Responses associated with the shearing of sands (Leonards et al, 1995).

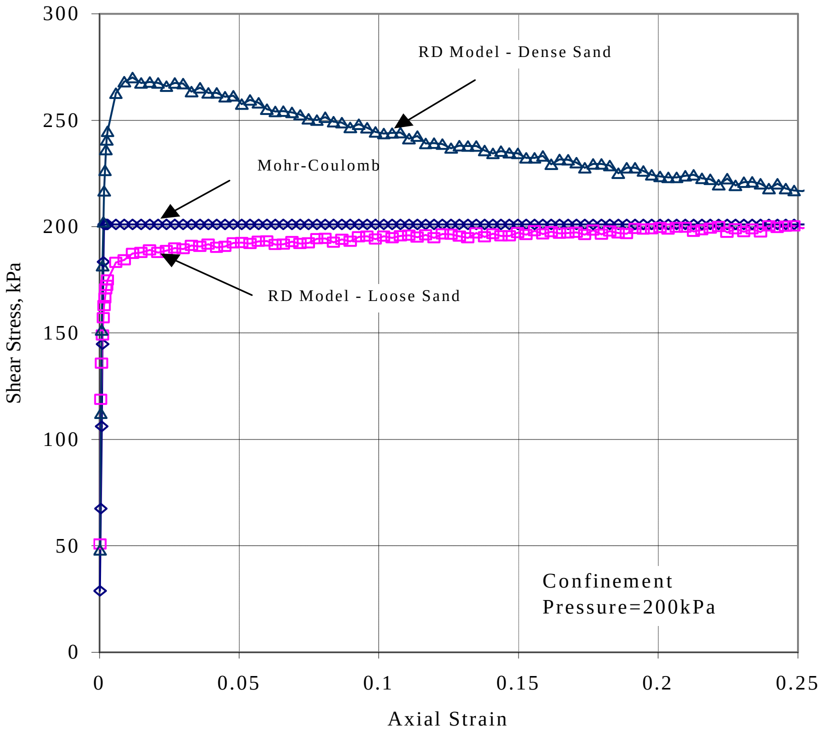

Figure 2 Stress versus strain curves of the triaxial compression tests on the soil models for the circular foundation simulations.

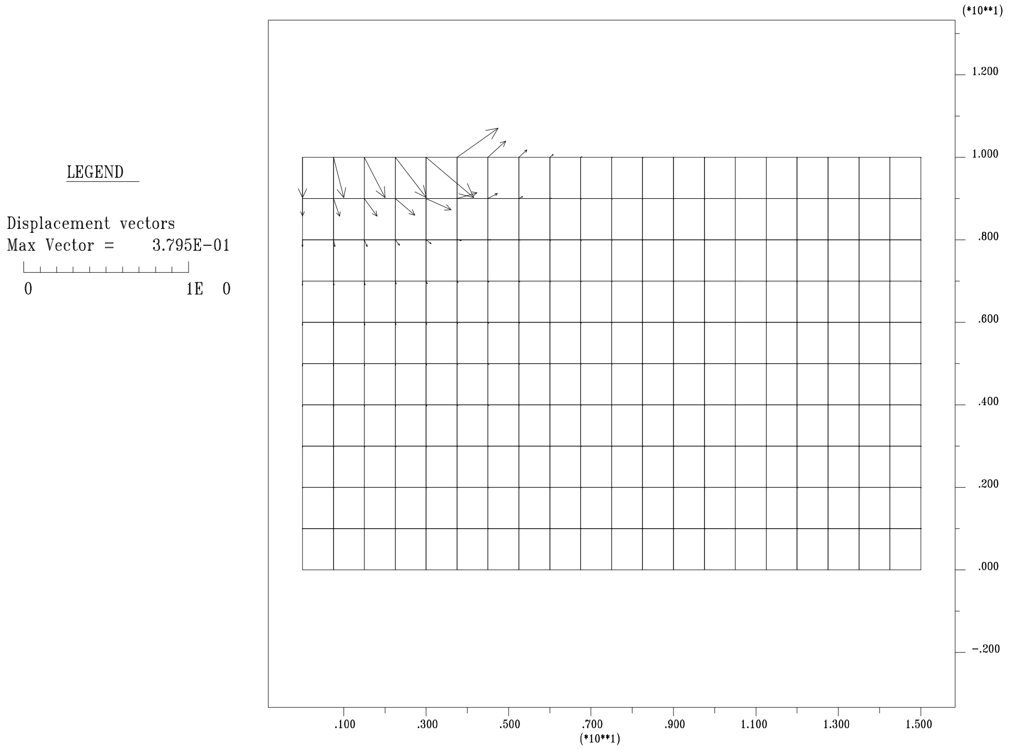

Figure 3 The axisymmetric mesh and the displacement vectors of the surface circular foundation using the Mohr-Coulomb model.

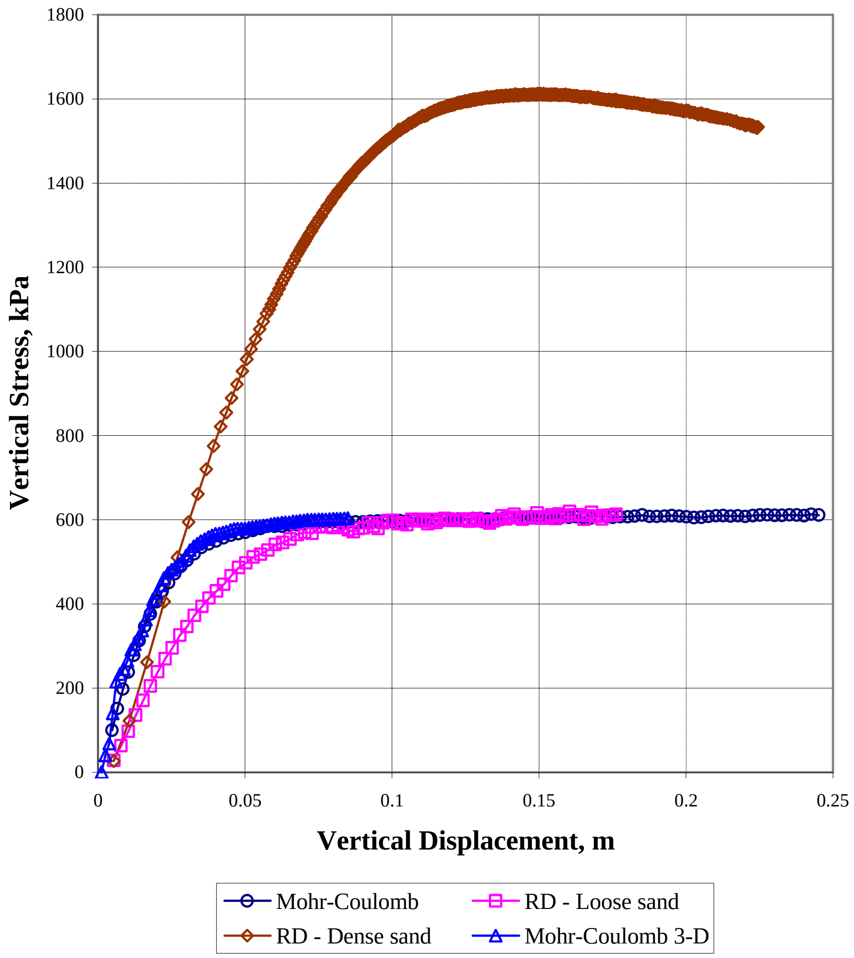

Figure 4 Vertical stress versus displacement of the circular foundation subjected to a vertical loading using the RD model and Mohr-Coulomb models.

6. Dynamic Critical State Behavior of Soils

It can be concluded from Chang and Whitman (1988) that when the void ratio is located above the critical state line, soil contracts while the shear strain accumulates with cyclic loading. When the void ratio is located below the critical state line, soil dilates while the shear strain accumulates with cyclic loading. Therefore, similar critical state behavior exists in cyclic loading. A considerable time was spent to incorporate a soil model similar to the RD model for dynamic loading conditions. The response of the model under controlled conditions is illustrated briefly in Figures 5 and 6.

Initially loose and dense sand elements were subjected to cyclic axial displacements while maintaining constant horizontal confining stresses. Figure 5 shows the displacement versus time of the elements of loose and dense sand subjected to a vertical cyclic displacement (δy) at 2 Hz. Negative displacement represents compression while positive denotes extension. The responses of lateral displacements (δx) versus time are shown. For the loose sand the values of δx are negative (contraction), while for the dense sand they are positive (dilation). The trend lines indicate that, given enough load cycles, the lateral displacements would approach a critical state condition in both cases.

Figure 6 shows the corresponding response of the generalized shear stress versus shear strain of the two soil elements. The generalized shear strains are related to the axial and lateral displacements shown in Figure 5. The loose sand shows densifications with each loading cycle.

Therefore, the shear stress required to maintain constant repetitive axial deformations increases with each successive cycle. While for the dense sands, the stress reduces with each cycle because the dense sand is loosening.

By integrating the area under the stress-strain curves, the energy loss per cycle can be obtained and this energy loss is called "material damping". The material damping varies with each cycle and it also varies with different stress paths. Successful incorporation of this model into FLAC would allow varying values of material damping to be accounted for cycle by cycle directly.

The results of the developed cyclic model under ideal conditions are qualitatively and quantitatively satisfying. However, it was not possible to develop a mean to detect stress reversal in a general stress state under complex loading conditions. By design, the explicit solution scheme in FLAC converges to a proper solution. The difficulty comes from the accelerations that take place during convergence (Deschamps, 1996). Inevitably, these accelerations would be interpreted as stress reversals and the complex system would develop errors. A robust scheme that could operate reliably under general states could not yet be identified. Therefore, when complicate analysis is necessary, such as pile driving, the model needs to be simplified (Feng, 1997).

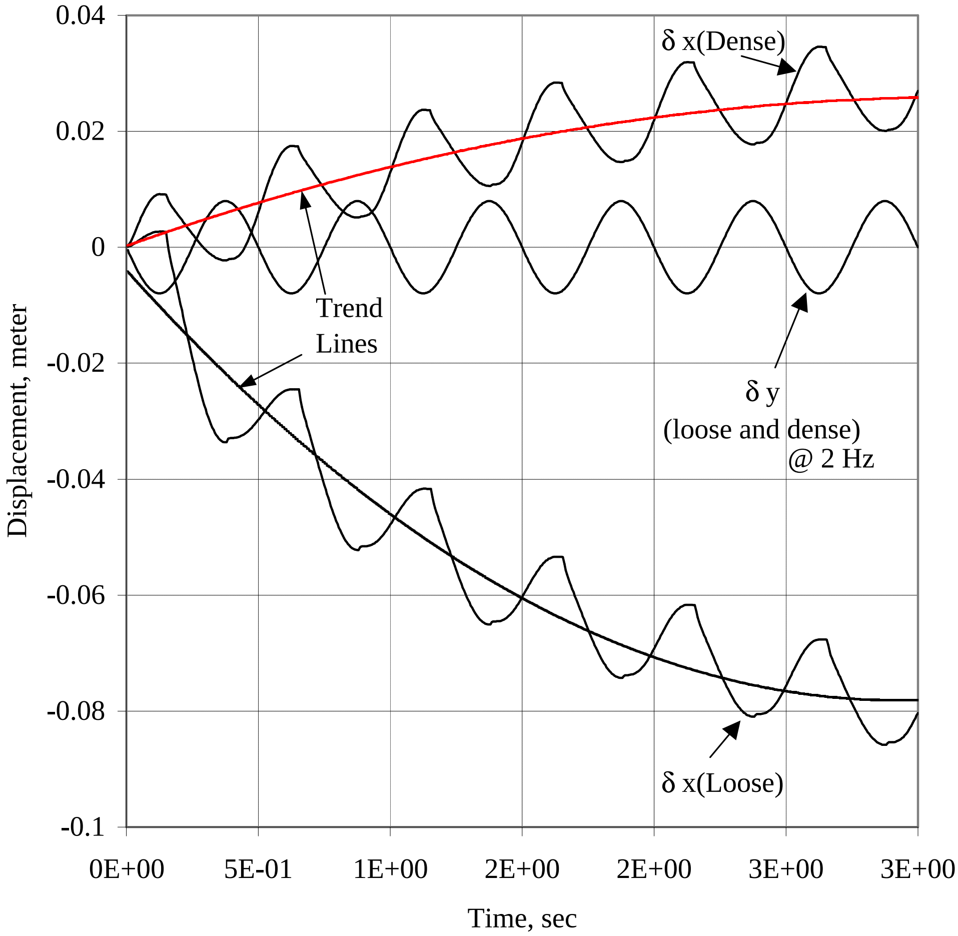

Figure 5 Lateral displacements (dx) versus time in response to the same imposed cyclic axial displacements (dy) for loose and dense soil elements (Leonards et al, 1995).

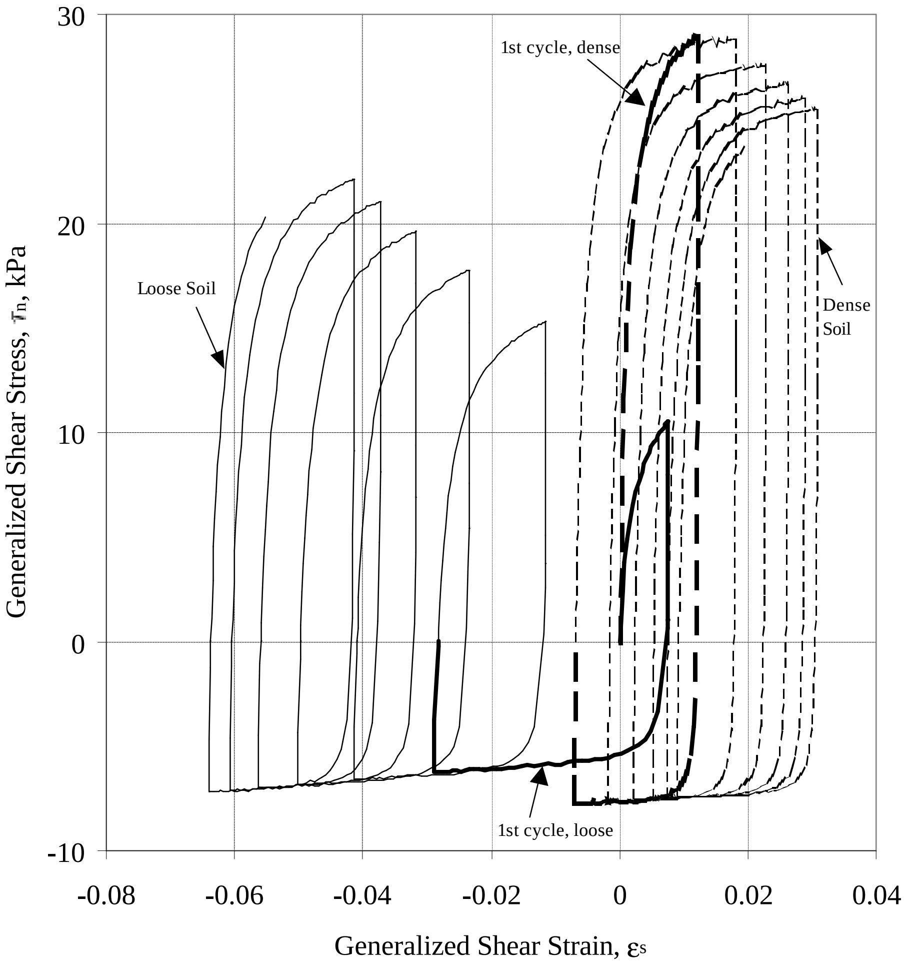

Figure 6 Shear stress-strain response of loose and dense soil elements subject to the same imposed cyclic axial displacements (*d*y)(Leonards et al, 1995).

7. Conclusion

A relative simple but realistic model, called RD model, was programmed using FISH language for cohesionless soils. The model can be applied to plane strain and axisymmetric problems and is based on the concept of critical state soil mechanics. It is capable of simulating dilative and contractive behavior of granular soils. The responses of a soil element subjected to an axial compression test were illustrated. Two conditions of the sands were simulated, that is, initially loose and dense states. The terms "loose" and "dense" are related not only to the void ratio of the soil but also to the magnitude of the stress level. In addition, an example of a circular surface foundation was simulated. The stress-strain results of the RD model for the foundation were compared to that of Mohr-Coulomb model under the same loading conditions. The results of the stress versus displacement of the simulations are presented and good agreement was achieved.

An attempt was made to simulate cyclic behavior of cohesionless soil, and the process was successful for controlled cyclic loading of laboratory tests. More development of the cyclic model is needed to fully implement the concept of the critical state in complicate dynamic simulation of soils.

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應用 FLAC 程式模擬砂性土壤的組成律

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摘要

本文旨在發展一個簡單但接近現實的土壤模式來模擬砂性土壤的反應，並可套用於 FLAC **程式中進行多種大地工程的分析。此土壤模式稱為〝**RD **模式〞，可使用在平面應變與軸對稱的問題中。此模式考慮土壤力學之臨界狀態**(critical state soil mechanics)**觀念且可模擬砂性土壤的收縮與膨脹行為。此模式的發展使用了** FISH **語言。**FISH **語言為** FLAC **程式中用來控制分析的特殊語言，且可允許使用者建立自定的土壤模式。自定的土壤模式撰寫後若編譯成功，其使用就如內建的土壤模式，但運算時較慢。**

為了解釋 RD **模式的行為，本文分別模擬了鬆砂與緊砂在三軸壓縮試驗中的反應。土壤之疏鬆與緊密狀態不僅與孔隙比有關，且與應力狀態有關。與臨界曲線距離相關的狀態參數**(state parameter)**控制了土壤收縮與膨脹的量與速率，以及應力應變之關係曲線。模擬結果以應力路徑及應力應變曲線等來說明。**

為了示範 RD **模式的應用，本文模擬一個地表圓形基礎的載重反應。模擬的土壤條件有三個，即** RD **模式鬆砂狀態，**RD **模式緊砂狀態與** Mohr-Coulomb **模式。此地表圓形基礎模擬時假設為軸對稱，且基礎與土壤接觸面為平滑。分析所得的結果以應力與變位曲線表示出各種土壤條件下承載力之關係。**

土壤受到反覆載重時，可能也存在著一種臨界狀態。因此，本文亦嘗試著去發展一個動力土壤模式，來模擬砂性土壤受到反覆載重時之行為。模擬結果亦顯示，疏鬆狀態的砂土受到反覆載重時會緊密化，而緊密狀態的砂土則會疏鬆化。在理想的條件下，此土壤模式模擬結果應屬滿意，但想完全地將土壤臨界狀態之觀念應用於更複雜的土壤動力分析，則此模式仍需要做更進一步的修改與發展。

關鍵詞：土壤模式模擬、反覆載重模式、臨界狀態、砂性土壤、表面基礎、數值模擬、 FLAC