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Coupons Versus Rebates

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This paper examines a key difference between two promotional vehicles, coupons and rebates. Whereas coupons offer deals up front, with the purchase of the product, rebates can be redeemed only after purchase. When consumers experience uncertain redemption costs, this difference translates to a difference in when uncertainty is resolved. With coupons the uncertainty is resolved before purchase; with rebates the uncertainty is resolved after purchase. As a result, we show that rebates are more efficient in surplus extraction but coupons offer more finetuned control over whom to serve.

We identify the conditions under which each is optimal, and these conditions turn on the gap between "low" reservation price consumers' valuations and their highest redemption costs. Rebates are optimal when this gap is large; coupons tend to be optimal otherwise. Risk aversity on the part of consumers reduces the attractiveness of rebates, as does the delay between rebate redemption and rebate payment, but the latter if and only if consumers are more impatient than the seller. These observations match up well with what we know about the use of these promotional vehicles in the real world.

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1. Introduction

The idea that a seller can use promotional vehicles like coupons and rebates to price-discriminate among consumers is a commonplace by now. In fact, it is so commonplace that it is the stuff of undergraduate textbooks. Pindyck and Rubinfeld (1995), in their *Microeconomics*, explain it this way:

> Coupons provide a means of price discrimination. Studies show that only about 20 to 30 percent of all consumers regularly bother to clip, save, and use coupons when they go shopping. These consumers tend to be more sensitive to price than those who ignore coupons. They generally have more price-elastic demands and lower reservation prices. So by issuing coupons, a cereal company can separate its customers into two groups, and in effect charge the more price-sensitive customers a lower price than the other customers.
>
> Rebate programs work the same way.

This paper is about the last sentence. Do coupons and rebates really work the same way? In the academic literature, certainly, the presumptive answer seems to be "yes"—perhaps an unfortunate consequence of the easy recognition that coupons and rebates can both be used as price discrimination tools. Pindyck and Rubinfeld are not alone in lumping the two together. The following passage is from Gerstner and Hess (1991):

> This distinction between a coupon, which is a certificate that gives an immediate price reduction upon purchase of the product, and a rebate, which is a refund sent to the buyer after the purchase, is immaterial if consumer redemption costs are identical. Here, "rebate" is used interchangeably for "rebate or coupon" to improve readability.

Blattberg and Neslin (1990, p. 13) describe rebates as the "durable goods analog" of coupons. The models in Levedahl (1984), Narasimhan (1984), Shaffer and Zhang (1995), and Moraga-Gonzalez and Petrakis (1999) can be interpreted as pertaining to coupons or rebates.

We wish to argue in this paper that coupons and rebates do not work the same way even if their redemption costs are identical, and these differences have substantive implications for when each is optimal. The difference we focus on is the one embedded in the quotation from Gerstner and Hess (1991) above, namely, that coupons are redeemed with the purchase, whereas rebates are redeemed after the purchase. This means that if someone responds to a coupon, then she pays a lower price—the promotional price. However, with rebates it is possible to respond to the rebate and still pay *regular* price. This is because consumers often do not redeem rebates, even if their original motivation for buying the product was the rebate (Dhar and Hoch 1996, Hoch et al. 1997). Indeed, in the rebate-fulfillment industry, it is taken as a given that this "slippage" is what makes rebates so attractive as promotion vehicles (Bulkeley 1998, Edmonston 2001, Spencer 2002). For example, Spencer (2002) observes:

> Of course, to some degree companies that offer mail-in rebates have always banked on your laziness. Only 5% of offers are claimed by consumers, which allows companies to advertise low prices without having to absorb the cost of a flat-out price cut. Claim rates rise with the value of the rebate, but even on electronics, redemption rates are still only 40% on average, according to BDS Marketing.

In fact, the slippage may be so high that it is possible for sellers to make "free-after-rebate" offers.[1]

It would seem that rebates hold all the cards from the seller's point of view. Sellers collect the regular price up front, and incur a promotional cost only if the consumer redeems the rebate. The risk of nonredemption is borne by the consumer. By contrast, with coupons, if the coupon is successful in motivating a purchase by the marginal consumer, then the seller incurs the promotional cost up front. If the coupon is not successful in motivating a purchase, then the seller has lost the sale.

While this analysis may point to rebates as the optimal solution, the reality is that coupons exist—in large numbers. According to NCH Marketing Services, in 2002, 248 billion manufacturer coupons were distributed by consumer packaged-goods companies in the United States, and over $3 billion in coupon value redeemed.

What might be the reasons marketers use both coupons and rebates? Under what circumstances are coupons optimal, and under what circumstances are rebates optimal? While no formal analysis of these questions appears in the literature, informal answers are readily given. For example, rebates are simply too costly to redeem for frequently purchased low-ticket items, which typically offer small discounts. The average coupon discount offered on packaged goods in 2002 was 89 cents; it would be hard to imagine a consumer putting in the effort to redeem a mail-in rebate of this size, considering that it costs 37 cents to mail a first-class letter in the United States. However, this begs the question, "Why aren't coupons used more often for high-ticket products?"[2] For this, too, an institutional reason can be given. Coupons are more prone to fraud than rebates, and fraud is costlier when the amounts involved are larger. These "negative justifications" are not entirely satisfactory, however. Institutional constraints are not immutable. As technology evolves, one may imagine different ways of redeeming coupons and rebates developing. If the institutional constraints are whittled away, would rebates be offered for low-ticket products and coupons for high-ticket products? It would seem that there is a need for a theory that examines the inherent differences between these promotion vehicles. This paper provides such a theory. As we discuss in the conclusion, a key benefit of the theory is that it enables a broader view of coupons and rebates, one in which the traditional notions of what is a coupon and what is a rebate are no longer valid.

The existing literature focuses on coupons or rebates, not both. For example, there is a large literature on coupons, dealing with a multitude of issues: use of coupons for market research (Nielsen 1965), targeted price promotions (Robinson 1977), how coupon users differ from nonusers (Narasimhan 1984), and the differences among different types of coupons—direct-mail coupons (Neslin and Shoemaker 1983), package coupons (Raju et al. 1994, Dhar et al. 1996), cross-ruff coupons (Dhar and Raju 1998), short versus long-duration coupons (Krishna and Zhang 1999), and front-loaded versus rear-loaded coupons (Zhang et al. 2000). The literature on rebates seems to suggest that only rebates ought to be used. Chen et al. (2005), for instance, argue that only rebates have the capability of providing state-dependent discounts after the purchase. Because coupons cannot do that, they are presumably inferior. The behavioral literature, by focusing on the greater likelihood of "irrational" consumer behavior with rebates, implicitly endorses rebates. Hoch et al. (1997) observe that consumers may "space out" in the interval between when they are exposed to the promotion and when they have to redeem it. Such spacing-out behavior benefits the firm, even more so with rebates, because with them it happens after the consumer has paid regular price. Alternatively, this literature argues that consumers systematically misperceive the benefit-cost calculus of rebates at the time of purchase, accentuating the former, attenuating the latter (Akerlof 1991, Kahneman and Lovallo 1993, Loewenstein 1996, Soman 1998). Then, when the actual redemption decision is faced, redemption costs loom larger than originally thought, and rewards do not seem as attractive as before. Presumably, these seller-favoring misperceptions and

[1] A Google search on "free-after-rebate" produced 76,400 results on October 21, 2004 at 4:15 PM (EDT). The first result is a site called "Free After Rebate," whose motto is "The Best Things in Life Are Free (After 6 to 8 Weeks)."

[2] While it is commonly believed that coupons are only offered for low-ticket, frequently purchased goods, this belief is based largely on the experience of manufacturer coupons. Retailer coupons are more diverse. For example, online retailers like Dell offer coupons even for high-ticket, durable goods. In addition, bricks-and-mortar retailers selling high-ticket services like health clubs, restaurants, etc., offer high-value coupons.

dynamic inconsistencies are not present to the same extent with coupons, so this line of argument, barring institutional constraints, would also endorse rebates over coupons.

Our explanation of the choice between coupons and rebates does not rely on forgetful consumers or consumers misperceiving their decision problem. Consumers in our model are fully rational—they correctly perceive the costs and benefits of promotions and redeem them only if they are more valuable than the redemption cost—and we do not put any restrictions on the use of either promotion vehicle. In fact, but for the definitional difference between them, coupons and rebates are interchangeable in our model. Still, we find that coupons are optimal sometimes and rebates are optimal at other times. This difference in outlook doesn't mean that we deny the possibility of forgetful consumers or consumer misperceptions. Indeed, our model of uncertain redemption costs can be seen as a way to capture forgetfulness in a rational-consumer framework. Our point is that even after consumers have been sensitized to their forgetfulness and their tendency to be excessively optimistic about their rebate redemption behavior, there are still reasons to believe that coupons are optimal under some circumstances and rebates in others.

The starting point of our model is the recognition that not all promotions are redeemed, and redemptions vary between consumers and within consumers. For instance, NCH Promotional Services estimates that in 2002, 18.5% of shoppers always used coupons, 37.6% "sometimes" used coupons, 23.3% "rarely" used coupons, and 20.6% never used coupons. We capture this variability by assuming that promotion redemption costs are stochastic and heterogeneous. That coupons and rebates impose redemption costs on the consumer is straightforward; it is the basis of their price discrimination capability. In the case of coupons, these costs include the costs of finding them (for example, in the United States, coupons are generally offered in freestanding inserts in the Sunday newspaper, which require a consumer to have a newspaper first and the willingness to peruse it to find a coupon of interest), clipping it, and the costs of remembering to produce it on the next shopping trip (before it expires).[3] With rebates, too, there are the costs of finding them and the costs of remembering to redeem them before they expire, but also there are the costs of fulfilling the redemption requirements—for example, most rebate offers require filling out a form, enclosing a UPC code from the product package, and mailing in the rebate form—and the costs of waiting for the rebate check to arrive. These structural, built-in redemption costs of each promotion vehicle get translated into the consumer's personal terms, producing variability in effective redemption costs, across and within consumers. For instance, different consumers might have different redemption costs because they differ in their opportunity cost of time (e.g., high-income versus low-income consumers; families with young kids versus single-member households). In addition, a given consumer may experience different redemption costs at different times because of random factors: Sometimes the consumer is busy (referee-report due, relatives visiting, children sick), and sometimes she has more time.

Going into a purchase, therefore, a consumer is likely to experience uncertain redemption costs with both coupons and rebates. Our analysis focuses on when this uncertainty is resolved relative to purchase in the two cases. With coupons, the uncertainty about redemption costs is resolved before the consumer has bought the product; with rebates, the uncertainty is resolved after she has bought the product (at the regular price). From the seller's perspective, this translates to another sort of difference: With rebates, all consumers with a given reservation price are ex ante (i.e., at the time of purchase) identical to the seller; with coupons, all consumers with a given reservation price are not ex ante identical to the seller—the seller knows that among them are people who have experienced high redemption costs and others who have experienced low redemption costs. We show that this difference between the two promotion vehicles has interesting implications for how the offer appears to the consumer at the time of purchase, which in turn confers different advantages and disadvantages to them. Rebates unbundle the redemption decision from the purchase decision, but in the process they present a bundled offer to the consumer: Her only options are to buy the product or not to buy. Coupons, on the other hand, bundle the redemption and purchase decisions, but this unbundles the offer: The consumer can buy or not buy the product, with or without the coupon.

These differences translate to different properties for the two promotion vehicles. Consumers who buy only because of the coupon can never "space out" and not redeem the coupon in our model; with rebates, this is possible. A mean-preserving spread in redemption costs has no effect on the promotional price under rebates, but it does under coupons. In equilibrium,

[3] If a manufacturer coupon redeemable at a bricks-and-mortar store is offered online, then the redemption cost includes the cost of going to the online site—which is likely to be an added cost because the online site, unlike the Sunday newspaper, doesn't necessarily "come" to the consumer—the cost of perusing the website, the cost of printing the coupon, and then, again, the cost of remembering to produce it on the next shopping trip (before it expires). In the case of coupons offered by online retailers, the latter two cost categories are eliminated because such coupons only require a coupon code to be entered at checkout.

the redemption rate on rebates is positively associated with rebate value, both covarying with redemption costs, but the relation between coupon redemptions and coupon face value is ambiguous, depending not only on redemption costs but also on product valuations. This last observation provides theoretical support for Neslin and Clarke's (1987) empirical finding of a lack of relationship between absolute coupon value and redemption rate in cross-sectional coupon data. It also suggests that relative coupon face value—coupon value as a percentage of regular price—is probably the better variable to use in such studies, a conjecture confirmed in Reibstein and Traver (1982), who find a positive relationship with it.

Rebates price discriminate more efficiently than coupons among those who buy the product, but coupons offer more finetuned control to the seller in who gets to buy the product. We show that the resulting trade-offs lead to coupons being optimal only when the least attractive segment is not very attractive—when the difference between its product valuation and redemption cost is small (as with low-ticket goods). Rebates are optimal when it is optimal to serve even the least attractive segment (as with high-ticket goods). Risk aversion on the part of consumers weakens rebates, because such consumers demand compensation up-front, in the regular price, for the risk of not redeeming the rebate. Rebates are unable to provide targeted relief in mitigating this risk compensation: The seller is forced to serve an entire consumer type, or not serve them at all. Delay between rebate redemption and rebate payment also hurts rebates, but if and only if consumers have a lower discount factor than the seller. These results match up well with what we know about the usage of coupons and rebates in the real world.

2. Model

Consider a seller facing a market with two types of consumers, h and l, of sizes α and $1-\alpha$, respectively. Consumers of type h have a higher reservation price for the seller's product than consumers of type l ($V^h > V^l$). The marginal cost of production is constant and can be normalized to zero by interpreting the reservation prices as net of marginal cost.

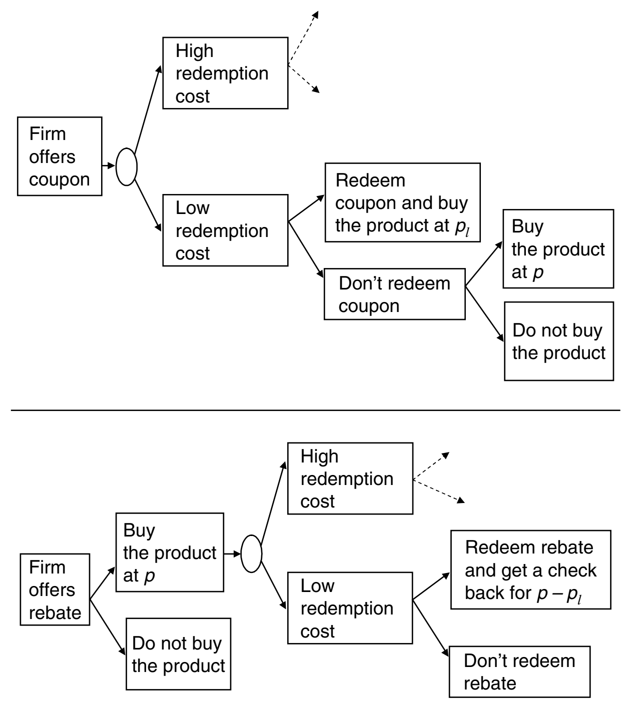
The seller would like to charge V^h to consumers of type h and V^l to consumers of type l, but of course, it cannot directly discriminate between different consumers based on their identity. Instead, it offers a promotion, either as a coupon or as a rebate, and allows consumers to self-select whether to pay the regular price p, or the promotional price p_l, or even whether to buy the product at all. In designing the promotion, the seller takes into account consumers' reservation prices, type sizes, redemption costs, and other relevant aspects of the consumer environment as described below.

As noted in the introduction, we assume that consumers' redemption costs are the same for coupons and rebates. The question of which of them is optimal will not, therefore, turn on which is costlier to redeem. Consumers are heterogeneous in their redemption costs and face uncertain redemption costs with either promotional vehicle. The latter property we model as consumers finding themselves in one of two "states of nature:" a high-redemption-cost state ("high state") or a low-redemption-cost state ("low state"). For example, on the day the coupon arrives, a consumer might find herself very busy. Alternatively, after taking the rebate form home on a Saturday, she might find herself in a state of considerable leisure.[4] Let $\bar{C}^h$ and $\underline{C}^h$ denote the high and low redemption costs of consumers of type h, and $\bar{C}^l$ and $\underline{C}^l$, the corresponding quantities for type-l consumers. Let λ be the probability of the high state for either segment, and we assume, naturally, that the occurrence of these states is independent across consumers. We shall assume that $V^h > \bar{C}^h$ and $V^l > \bar{C}^l$, so that both segments can potentially be served promotions. Consumers of type h have a higher expected redemption cost for either promotional vehicle than consumers of type l. That is, $E(C^h) = \lambda\bar{C}^h + (1-\lambda)\underline{C}^h > \lambda\bar{C}^l + (1-\lambda)\underline{C}^l = E(C^l)$. This sort of assumption is natural, and has appeared many times before in the literature, for example, in Narasimhan (1984), Jeuland and Narasimhan (1985), Iyer (1998), and Banks and Moorthy (1999). It is easily motivated by noting that both reservation price and redemption costs are driven by the same underlying consumer characteristics, for example, income. Higher-income consumers are willing to pay higher prices than lower-income consumers and have higher redemption costs as well because of their greater opportunity cost of time.

The crucial difference between coupons and rebates is, of course, the different sequence of decisions in the

[4] Our notion of "uncertain redemption costs" should really be seen as a *modeling device* that captures summarily what we know to be true—that consumers sometimes redeem coupons/rebates and sometimes they do not. In practice there is an expiry date that is several weeks away from when a coupon/rebate first appears, and a consumer may experience several bouts of high and low redemption costs during this period. One might argue that she has the luxury of postponing her redemptions to the nonbusy periods. Our formulation may be seen as a reduced-form representation of this optimal timing problem: We simply reinterpret our redemption cost as the "lowest redemption cost before the expiry date." In other words, the "lowest redemption cost before the expiry date" itself is stochastic. This way, our model captures "forgetfulness" and ties it to the expiry period. While a long expiry period allows the consumer to override moments of busyness, it also encourages procrastination and forgetting ("out of sight, out of mind").

Figure 1 **Sequence of Consumer Decisions: Coupons Versus Rebates**



two cases, and when uncertainty is resolved in relation to those decisions. These are depicted in Figure 1. In the case of coupons, the consumer's redemption-cost uncertainty is resolved before she buys the product; depending on the realization, either she will decide to redeem the coupon or not. If she decides to redeem the coupon, then of course, she will also buy the product; but if she decides not to redeem the coupon, then she might or might not buy the product, depending on whether the regular price is attractive enough (relative to her reservation price). In the case of rebates, the purchase decision comes first, then the rebate-redemption decision.[5] When the consumer makes the buying decision, she does not know whether she will experience low or high redemption costs in the future, so she does not know whether she will end up redeeming the rebate or not. Being rational, she will anticipate this uncertainty and make the buying decision on an *expected* utility basis. If she chooses to buy, then subsequently she will experience either high or low redemption costs, and depending on the realization, she will either redeem the rebate or not. In short, in the case of coupons, despite variable redemption costs, the consumer's payoffs are certain at the time of the coupon-redemption/buying decision. With rebates, these payoffs are uncertain at the time of the buying decision.

To highlight the role of these different sequences of decision making and uncertainty resolution in how these two promotion vehicles work, we set things up so that their effects can be determined cleanly and transparently. We first assume that consumers are risk neutral and that there is no delay between rebate redemption and rebate payment; later we discuss the effect of relaxing these assumptions. With these assumptions, plus our assumption that redemption costs are the same between the two promotion vehicles, everything is the same between the two except the basic structural difference in the way they work—something we cannot equalize away without destroying the very nature of these tools.

3. Analysis

The seller first has to decide whether to offer a promotion or not and, given a promotion, whether to offer it as a coupon or as a rebate. Our analysis proceeds in the reverse direction. We first analyze the various coupon options to decide on the optimal coupon, then the various rebate options to decide on the optimal rebate, and then compare the best coupon option with the best rebate option—all the while assuming that a promotion is required. Last, we ask whether the seller should promote at all, given the optimal promoting strategy if it does promote. Not promoting would mean either an "everyday high price" (V^h) or an "everyday low price" (V^l). This introduces another constraint, and in Proposition 4 we integrate the entire analysis, providing the necessary and sufficient conditions to promote with coupons, rebates, or not promote at all.

Note first of all that without uncertainty, coupons and rebates lead to the same outcomes in our model, and the seller ought to be indifferent between the two promotion vehicles. For example, suppose $\bar{C}^h = \underline{C}^h = C^h > C^l = \bar{C}^l = \underline{C}^l$ and $V^h - V^l \geq C^h - C^l$. Then, the optimal coupon solution is to offer a promotional price $V^l - C^l$ and a regular price $(V^l - C^l) + C^h - \epsilon$ (where $\epsilon > 0$ can be arbitrarily small). Type-h consumers pay the regular price and type-l consumers pay the promotional price, redeeming a coupon. With a rebate, as well, the same outcomes are optimal, except now both consumer types pay $(V^l - C^l) + C^h - \epsilon$ up-front, but only the type-l consumer ends up redeeming the rebate—a check worth $C^h - \epsilon$. In other words, the timing difference between when coupons are redeemed and when rebates are redeemed, by itself, does not have any material impact on the seller's fortunes. What is also needed is uncertainty to interact with the timing difference.

[5] Coupons and rebates, the way we are modeling them, bear some resemblance to the notions of "spot selling" and "advance selling," respectively, in Shugan and Xie (2000), Xie and Shugan (2001), and Biyalogorsky and Gerstner (2004). For example, offering a rebate may be seen as analogous to advance selling, and coupons to spot selling. However, even when advance selling is implemented, spot selling is always available while coupons and rebates are rarely offered together; nor do we recommend that they should be.

To make the exposition easier, we assume that $V^h - V^l \geq \bar{C}^h - \underline{C}^l$ and $\bar{C}^h > \underline{C}^h > \bar{C}^l > \underline{C}^l$, a further specialization of the assumptions $V^h > \bar{C}^h$, $V^l > \bar{C}^l$, and $E(C^h) > E(C^l)$; in the appendix we verify that these restrictions are not material to our qualitative conclusions.

3.1. Optimal Coupons

In designing the optimal coupon, the seller faces the classical price-discrimination dilemma: how to design the coupon so that it can charge each consumer type close to its reservation price while preventing the high-reservation-price consumers from using the lower price. In the present instance, it is helpful to think of the market as really consisting of four segments: (1) $(V^h, \bar{C}^h)$, (2) $(V^h, \underline{C}^h)$, (3) $(V^l, \bar{C}^l)$, and (4) $(V^l, \underline{C}^l)$, with a priori sizes $\alpha\lambda$, $\alpha(1-\lambda)$, $(1-\alpha)\lambda$, and $(1-\alpha)(1-\lambda)$, respectively.

The coupon-redemption decision for a consumer involves two calculations: (i) whether or not the coupon is worth redeeming considering the redemption cost, and (ii) whether or not the promotional price is below the consumer's *net* reservation price for the product (reservation price minus realized redemption cost). On the first criterion, it is straightforward, given $\bar{C}^h > \underline{C}^h > \bar{C}^l > \underline{C}^l$, that if type-$h$ consumers in their high-redemption-cost state prefer the coupon to the regular price, i.e., $p - p_l \geq \bar{C}^h$, then so will consumers of any other type or state. However, the second criterion is not necessarily fulfilled in the same way (for instance, $V^h - \underline{C}^h \geq p_l$ does not imply $V^l - \bar{C}^l \geq p_l$). It is obvious that the optimal coupon will not involve a discount so high that both types want to redeem it in both redemption states (because if this were so, the seller would prefer to offer everyday low prices). It is also obvious that the coupon discount cannot be so small that even type-l consumers in their low redemption state choose not to redeem (because if this were so, the seller would prefer to offer everyday high prices).

The main analytical problem to be solved is where to draw the boundary for which consumer segments will redeem the coupon. Anybody not using the coupon can buy at the regular price, or not buy at all. This leads to three main options and some suboptions:

It is immediate that option (c3a) is suboptimal. If the seller lets type-l consumers in the high state purchase the product without coupons, then the highest regular price possible is their reservation price V^l, and so the highest promotional price is $V^l - \underline{C}^l$. But this is definitely worse than eliminating the coupon and just charging a regular price V^l. This leaves us with four options to consider further.

Table 4 (in the appendix) summarizes the problem formulation in each case and the corresponding solutions. In each case, the promotional price is fixed first, as the highest price that can be charged the marginal consumer using the coupon. The regular price is then *built* on top of this promotional price by adding the coupon face value to the promotional price, with the coupon face value set so that the coupon is *not* redeemed by the marginal consumer using the regular price. To illustrate, consider option (c1a). Who is the marginal consumer using coupons? Because $V^h - V^l \geq \bar{C}^h - \underline{C}^l$ implies $V^h - V^l > \underline{C}^h - \bar{C}^l$, the marginal consumer is the type-l consumer in the high state. Thus, the highest promotional price the seller can charge is $p_l = V^l - \bar{C}^l$. Who is the marginal consumer paying regular price? It is the type-h consumer in the high state. So the optimal regular price is $p = p_l + \bar{C}^h = V^l + (\bar{C}^h - \bar{C}^l)$. We can verify that this price is smaller than V^h (because $V^h - V^l \geq \bar{C}^h - \underline{C}^l$), and at the same time satisfies the requirement that the coupon discount, $\bar{C}^h$, is not more than this segment's redemption cost.

Table 1 shows that no single option dominates the others: It is possible to find conditions under which each option is optimal. The choice among them turns on a two-dimensional comparison. One dimension is defined by whether or not $\lambda(1-\alpha)(V^l - \underline{C}^l) \geq (\bar{C}^l - \underline{C}^l)$, which determines whether the promotional price is "low," $p_l = V^l - \bar{C}^l$, or "high," $p_l = V^l - \underline{C}^l$. The other dimension is defined by whether or not $\underline{C}^h \geq \lambda\bar{C}^h$, which determines whether the coupon face value is "small," $p - p_l = \underline{C}^h$ or "large," $p - p_l = \bar{C}^h$.

Proposition 1. *In the optimal coupon program,*
- *The promotional price is determined by whether $\lambda(1-\alpha)(V^l - \underline{C}^l) \geq (\bar{C}^l - \underline{C}^l)$ or not. If $\lambda(1-\alpha)(V^l - \underline{C}^l) \geq (\bar{C}^l - \underline{C}^l)$, then $p_l = V^l - \bar{C}^l$; otherwise $p_l = V^l - \underline{C}^l$.*

Segment	Size	Option c1a	Option c1b	Option c2	Option c3a	Option c3b
1. $(V^h, \bar{C}^h)$	$\alpha\lambda$	Reg. price	Reg. price	Reg. price	Reg. price	Reg. price
2. $(V^h, \underline{C}^h)$	$\alpha(1-\lambda)$	Coupon	Coupon	Reg. price	Reg. price	Reg. price
3. $(V^l, \bar{C}^l)$	$(1-\alpha)\lambda$	Coupon	Don't buy	Coupon	Reg. price	Don't buy
4. $(V^l, \underline{C}^l)$	$(1-\alpha)(1-\lambda)$	Coupon	Coupon	Coupon	Coupon	Coupon

Table 1 Optimal Coupons

	$\lambda(1-\alpha)(V^l-\underline{C}^l) \geq (\bar{C}^l-\underline{C}^l)$	$\lambda(1-\alpha)(V^l-\underline{C}^l) < (\bar{C}^l-\underline{C}^l)$
$\underline{C}^h \geq \lambda\bar{C}^h$	Promo price: $V^l-\bar{C}^l$ (low); Coupon value: $\underline{C}^h$ (small); Redemptions: $1-\alpha$ Profit: $V^l-\bar{C}^l+\alpha\underline{C}^h$ (option c2)	Promo price: $V^l-\underline{C}^l$ (high); Coupon value: $\underline{C}^h$ (small); Redemptions: $(1-\alpha)(1-\lambda)$ (lowest) Profit: $(V^l-\underline{C}^l)[1-\lambda(1-\alpha)]+\alpha\underline{C}^h$ (option c3b)
$\underline{C}^h < \lambda\bar{C}^h$	Promo price: $V^l-\bar{C}^l$ (low) Coupon value: $\bar{C}^h$ (large) Redemptions: $1-\alpha\lambda$ (highest) Profit: $V^l-\bar{C}^l+\alpha\lambda\bar{C}^h$ (option c1a)	Promo price: $V^l-\underline{C}^l$ (high); Coupon value: $\bar{C}^h$ (large) Redemptions: $1-\lambda$ Profit: $(V^l-\underline{C}^l)[1-\lambda(1-\alpha)]+\alpha\lambda\bar{C}^h$ (option c1b)

• *The coupon value is determined by whether $\underline{C}^h \geq \lambda\bar{C}^h$ or not. If $\underline{C}^h \geq \lambda\bar{C}^h$, then $p-p_l=\underline{C}^h$; otherwise $p-p_l=\bar{C}^h$.*

The first of the two conditions in the proposition can be written as $(V^l-\bar{C}^l)/(\bar{C}^l-\underline{C}^l) \geq (1-\lambda(1-\alpha))/\lambda(1-\alpha)$. The left-hand side is large when the gap between the reservation price of type-l consumers and their coupon-redemption cost in the high state is large, and the difference in coupon-redemption costs between the two states for these consumers is small; in other words, when both segments 3 and 4 are attractive. Then it pays to offer a promotional price low enough to allow all type-l consumers to buy the product at the promotional price. Conversely, the left-hand side is small when segment 3 is unattractive (for given V^l, $\bar{C}^l$ large), but segment 4 is relatively attractive ($\underline{C}^l$ small). Then the promotional price should be raised so that only type-l consumers in the low state use coupons. The standard for attractiveness is set by the ratio of the combined size of segments 1, 2, and 4 to the size of segment 3. The condition is natural because it captures the basic trade-off in choosing to serve type-l consumers in their worst state: Serving them implies a low promotional price (because the promotional price has to be low enough to cover their redemption cost), but a price this low means leaving "money on the table" on all other segments—including segments that buy at the regular price because the regular price has to be lowered to keep the coupon unattractive enough for these segments. Therefore, it may be better not to serve type-l consumers in their high state when they are relatively small in number, which translates to a small λ (high state unlikely) and/or a large α (type l small in number).

The regular price is determined by the coupon face value (given the promotional price), which in turn is determined by whether or not some type-h consumers should be allowed to redeem coupons. The advantage of letting low-state type-h consumers use coupons is that it enables a higher regular price—now only high-state type-h consumers have to be deterred from using the coupons, and this is easier because of their high redemption costs. The downside, of course, is that these consumers end up paying the promotional price when they could be paying the regular price. The trade-off is settled by looking at the redemption-cost difference between type-h consumers' high and low states; when the difference is small, it is difficult to separate these two types of segments, and the seller lets neither redeem the coupon. What is "small" is determined by the likelihood of the low state: The seller would be willing to let type-h consumers in the low state redeem the coupon, even with a small cost difference, when the low state is relatively unlikely.

Should coupon redemptions be maximized or minimized? Actually, neither need be the case. As Table 1 shows, expected coupon redemptions are maximized in option (c1a) and minimized in option (c3b), and both can be optimal, and neither need be optimal. The relationship between coupon value and expected redemptions is not one to one (see also Shaffer and Zhang 1995, 2002). In each panel of Table 1, expected redemptions increase as coupon value increases from top to bottom. However, going from left to right, horizontally, redemptions decrease while coupon value remains the same. If $\alpha < \lambda$, a diagonal move from (c2) to (c1b), due to a mean-preserving spread in type-h and type-l redemption costs, makes the coupon bigger but expected redemptions decrease. Why this ambiguity in the relationship between coupon redemptions and coupon value? The basic reason is, in deciding to redeem, a consumer considers not only coupon value relative to redemption costs, but also whether the product is worth buying. As a result, coupon redemptions in equilibrium depend on all parameters of the promotion environment, whereas coupon value

Table 2 Optimal Rebates

$\alpha\lambda\bar{C}^h \geq \max(\alpha\underline{C}^h, (\alpha+(1-\alpha)\lambda)\bar{C}^l)$	$\alpha\underline{C}^h \geq \max(\alpha\lambda\bar{C}^h, (\alpha+(1-\alpha)\lambda)\bar{C}^l)$	$(\alpha+(1-\alpha)\lambda)\bar{C}^l \geq \max(\alpha\underline{C}^h, \alpha\lambda\bar{C}^h)$
Promo price: $V^l-E(C^l)$	Promo price: $V^l-E(C^l)$	Promo price: $V^l-E(C^l)$
Rebate: $\bar{C}^h$ (highest)	Rebate: $\underline{C}^h$	Rebate: $\bar{C}^l$ (lowest)
Redemptions: $1-\alpha\lambda$ (highest)	Redemptions: $1-\alpha$	Redemptions: $(1-\alpha)(1-\lambda)$ (lowest)
$\pi=V^l-E(C^l)+\alpha\lambda\bar{C}^h$	$\pi=V^l-E(C^l)+\alpha\underline{C}^h$	$\pi=V^l-E(C^l)+(\alpha+(1-\alpha)\lambda)\bar{C}^l$
(option r1)	(option r2)	(option r3)

depends only on a subset of them. This provides theoretical support for Neslin and Clarke's (1987) inability to find a relationship between (absolute) coupon value and redemption rate in cross-sectional coupon data. It also suggests that "coupon %"—coupon face value as a percentage of regular price—is probably a better variable than absolute coupon value in such studies, a conjecture confirmed in Reibstein and Traver (1982), who use both variables and find a positive effect on redemption rate for both.

Note that optimal coupon face value does not respond to product valuations at all. If the high state is more likely, coupon value increases, as it becomes easier to prevent type-h consumers from using the coupon in their high state. Promotional price, on the other hand, responds to the type-l consumer's product value, her redemption costs, and relative segment sizes. Two types of effects can be identified. On the one hand, the promotional price itself is either $V^l - \bar{C}^l$ or $V^l - \underline{C}^l$, both of which are increasing in V^l, the product valuation. However, the discount off of V^l can be high or low depending on whether $(V^l - \bar{C}^l)/(\bar{C}^l - \underline{C}^l) \geq (1 - \lambda(1-\alpha))/\lambda(1-\alpha)$ or not. If this inequality is already satisfied and V^l increases, then the promotional price will increase, but if the increase in V^l produces a switch from $(V^l - \bar{C}^l)/(\bar{C}^l - \underline{C}^l) < (1 - \lambda(1-\alpha))/\lambda(1-\alpha)$ to $(V^l - \bar{C}^l)/(\bar{C}^l - \underline{C}^l) \geq (1 - \lambda(1-\alpha))/\lambda(1-\alpha)$, then the net effect is not clear. In general, this suggests that for high-ticket goods, not only is the promotional price likely to be higher, but the discount is also likely to be higher.

An increase in $\underline{C}^l$, for a given $\bar{C}^l$, has no effect on promotional price if the seller is already in the regime $(V^l - \bar{C}^l)/(\bar{C}^l - \underline{C}^l) \geq (1 - \lambda(1-\alpha))/\lambda(1-\alpha)$, otherwise it makes this regime more likely, which decreases the promotional price. An increase in $\bar{C}^l$, for a given $\underline{C}^l$, decreases promotional price as long as the seller is in the regime $(V^l - \bar{C}^l)/(\bar{C}^l - \underline{C}^l) \geq (1 - \lambda(1-\alpha))/\lambda(1-\alpha)$, but it makes a switch to the opposite regime more likely, which may cause an increase in the promotional price. In general, an increase in redemption costs decreases promotional price, as one would expect.

Changes in relative segment sizes due to α or λ determine only which regime occurs but do not affect the promotional price directly. In general, as the high state becomes more likely and/or the type-l segment becomes relatively larger, promotional prices are likely to be low and coupon value high. Empirically, this may correspond to the phenomenon of holiday promotions (Warner and Barsky 1995).

3.2. Optimal Rebates

Unlike the case of coupons, with rebates consumers are not sure whether they will redeem the rebate when they make the buying decision. They take their possible future behaviors into account when making the buying decision, but the buying decision itself cannot be contingent on future redemption states. Therefore, unlike the coupons case, here there are only two market segments at the buying stage: type h and type l. At the redemption stage, however, four market segments emerge just as in the coupon case, but unlike the coupon case, these segments are characterized by redemption costs only: (1) $\bar{C}^h$, (2) $\underline{C}^h$, (3) $\bar{C}^l$, and (4) $\underline{C}^l$. This is because the rebate-redemption decision depends only on whether the rebate check, $p - p_l$, will be large enough to cover the realized redemption cost. The purchase decision is sunk at this point, so product valuation does not enter the picture. The cost ordering $\bar{C}^h > \underline{C}^h > \bar{C}^l > \underline{C}^l$ sets a natural hierarchy: If a segment redeems, so will all lower segments in the hierarchy.

Obviously, a rebate policy in which type-l consumers do not buy the product cannot be optimal (because if that were so, an everyday-high-price strategy would be optimal and there would be no point in offering rebates). However, it is possible to design the rebate so that some consumers of a given type redeem the rebate—after experiencing low redemption costs—and others of the same type do not. Once again, the key issue is where to draw the boundary for which consumers, in which state, will redeem the rebate. There are three options:

Segment	Size	Option r1	Option r2	Option r3
1. $(V^h, \bar{C}^h)$	$\alpha\lambda$	Reg. price	Reg. price	Reg. price
2. $(V^h, \underline{C}^h)$	$\alpha(1-\lambda)$	Redeem	Reg. price	Reg. price
3. $(V^l, \bar{C}^l)$	$(1-\alpha)\lambda$	Redeem	Redeem	Reg. price
4. $(V^l, \underline{C}^l)$	$(1-\alpha)(1-\lambda)$	Redeem	Redeem	Redeem

Table 5 (in the appendix) summarizes the problem formulation in each case, and the respective solutions. All three options have the same promotional price, $p_l = V^l - E(C^l)$, because in each case the promotional price must be low enough to make the type-l consumers want to buy the product. However, the regular prices are different because the three options differ in who must be *prevented* from redeeming the rebate. In all options there is slippage: Some consumers do not redeem the rebate, at least sometimes. Slippage decreases with the rebate (we go from right to left in Table 2), which is intuitive. Note that in (r3), type-l consumers are marginal to the decision to buy the product—their expected consumer surplus from buying is exactly zero—and still they do not always redeem the rebate. In other words, it is possible for even marginal consumers to "space out" (Dhar and Hoch 1996, Hoch et al. 1997). With coupons this is impossible in our model: Type-l consumers in the low state in the right panel in Table 1 get zero consumer surplus, but they redeem the coupon.[6] In all rebate options, type-l consumers in the high state end up with negative consumer surplus: $V^l - (V^l - E(C^l)) - \bar{C}^l < 0$, which is impossible with coupons. In (r3), this happens despite not redeeming the rebate in the high state. They paid more than their reservation price at the time of purchase, counting on experiencing low redemption costs and redeeming the rebate. In the event, they experienced high redemption costs, when it became pointless to redeem the rebate (the purchase decision being sunk at this point).

By comparing the profits in the three options, it is easy to see that the choice among the three options turns on how $\alpha\lambda\bar{C}^h$, $\alpha\underline{C}^h$, and $(\alpha + (1-\alpha)\lambda)\bar{C}^l$ are ordered (see Table 2).

Proposition 2. *In the optimal rebate program,*

• *The promotional price upon redemption is* $p_l = V^l - E(C^l)$.

• *The rebate is* $p - p_l = \Delta$, *where* Δ *is* $\bar{C}^h$, $\underline{C}^h$, *or* $\bar{C}^l$ *depending on whether* $\alpha\lambda\bar{C}^h$, $\alpha\underline{C}^h$, *or* $(\alpha+(1-\alpha)\lambda)\bar{C}^l$ *is, respectively, the largest.*

The optimal rebate program is easier to characterize than the optimal coupon program. Instead of a two-dimensional characterization, now we have a simple comparison involving various redemption-cost magnitudes. The seller's problem is simpler because the consumer internalizes her uncertainty about redemption costs, unlike the coupons case, where the seller had to do so.

The main issue with rebates is how much the rebate should be, which directly plays into the issue of who the seller would want to redeem the rebate. Given the ordering $\bar{C}^h > \underline{C}^h > \bar{C}^l$, this becomes a classic margin-volume trade-off. Allowing more segments to redeem the rebate dilutes their margin contribution, but it also means that segments higher in the hierarchy can be charged a higher regular price. The conditions determining whether the rebate is "high," "medium," or "low" are based on a modified redemption-cost ordering, using weights to reflect these externalities. Each segment contributes a weight equal to its relative size (the product of its type size and state likelihood) plus the relative sizes of segments above it in the hierarchy. So, for example, $\underline{C}^h$ gets modified to $(\alpha(1-\lambda) + \alpha\lambda)\underline{C}^h = \alpha\underline{C}^h$. Slippage also depends on this modified redemption-cost ordering; in fact, there is a one-to-one inverse relationship between slippage and rebate size stemming from their common dependence on this ordering. (With coupons, by contrast, the relationship is looser; for instance, the same coupon may yield different numbers of redemptions depending on product value.) Simple heuristics such as "maximize slippage" or "minimize slippage" are clearly simplistic. For instance, (r2) has intermediate slippage, but when $\alpha\underline{C}^h \geq \max(\alpha\lambda\bar{C}^h, (\alpha + (1-\alpha)\lambda)\bar{C}^l)$, it is the optimal rebate program.

As in the case of coupons, the optimal promotional price increases in the value of the product and decreases in type-l consumers' redemption costs, but it depends on the latter only through *expected* redemption cost. Therefore, a mean-preserving spread in type-l consumers' redemption costs will not affect rebate-promotion price, whereas it does affect coupon promotional price. This reflects the fundamental timing difference between coupons and rebates that we have mentioned often in this paper.

The rebate itself (and hence the price actually paid) depends on redemption costs and relative segment sizes, but the latter only indirectly. Whether the rebate is "high," "medium," or "low," depends on which segment is marginal to the redeeming decision, which depends on the ordering of weighted redemption costs as noted above. If segment 2 is the marginal redeeming segment, then an increase in $\bar{C}^h$ (holding $\underline{C}^h$ and $\bar{C}^l$ constant) increases the rebate, but an increase in $\underline{C}^h$ (holding $\bar{C}^h$ and $\bar{C}^l$ constant) has no effect on rebate size as long as segment 2 continues to be marginal. However, if an increase in $\underline{C}^h$ makes segment 3 marginal, then rebate size will be bumped down to $\underline{C}^h$. $\bar{C}^l$ has the most leverage in this respect, followed by $\underline{C}^h$ and then $\bar{C}^h$. In general, as redemption costs increase, rebates increase, but clearly both means and variances matter. Mean-preserving spreads in redemption costs may increase, decrease, or have no effect on the rebate.

Finally, note that changes in α and λ may have different effects. For instance, the higher the probability of the high state, the lower the promotional price, *ceteris paribus*, but type-h size has no effect. The redemption rate, however, is decreasing in both, which is intuitive.

3.3. Coupons or Rebates?

We can analyze this question by framing the problem as one of achieving various behavioral outcomes, and asking whether coupons or rebates achieve them at a higher profit.

Type-h consumers pay regular price and type-l consumers pay promotional price.

This is option (c2) in the coupon case and option (r2) in the rebate case. With coupons, the seller sets

$$p_l = V^l - \bar{C}^l,$$
$$p = p_l + \underline{C}^h,$$

[6] In Dhar and Hoch's (1996) field experiment, they found evidence of spacing-out behavior even with shelf coupons. This is because in the real world there is a time interval between the purchase decision and coupon redemption. Because the time interval between the purchase decision and rebate redemption is even larger, spacing-out behavior by marginal consumers ought to be lower for coupons than for rebates.

and with rebates the seller sets

$$p_l = V^l - E(C^l),$$
$$p = p_l + \underline{C}^h.$$

The seller gets higher promotional and regular prices with rebates for the same consumer behavior. Clearly, rebates are better than coupons if this is the consumer behavior the seller wants to generate. Why is rebate superior in this case? With coupons, at the time of purchase, consumers know what state they are in, so the promotional price must be low enough to induce the high-state consumers of type l to buy. However, with rebates, consumers, when they purchase the product, do not know whether they will redeem the rebates or not, and they internalize this uncertainty in their buying decision. The seller has to respect this internalization when setting the regular price—otherwise type-l consumers will not buy—but it can take advantage of the possibility that low redemption costs can arise.

Type-h consumers in the high state pay regular price; all others pay promotional price.

This is option (c1a) in the coupon case and option (r1) in the rebate case. With coupons, the seller sets

$$p_l = V^l - \bar{C}^l,$$
$$p = p_l + \bar{C}^h;$$

but with rebates, it sets

$$p_l = V^l - E(C^l),$$
$$p = p_l + \bar{C}^h.$$

The seller achieves the same behavioral outcome in each case, but both the regular price and promotional price are higher with rebates. Therefore, rebates are better than coupons if the goal is to get type-h consumers in the high state paying regular price and all others paying the promotional price.

Type-h consumers pay regular price; type-l consumers pay the promotional price in the low state and do not purchase in the high state.

This is option (c3b) in the coupon case, and the seller cannot achieve this outcome with rebates. The conditions governing option-(c3b) optimality are $\lambda(1-\alpha)(V^l - \underline{C}^l) < (\bar{C}^l - \underline{C}^l)$ and $\underline{C}^h \ge \lambda\bar{C}^h$ (cf. Table 1). To complete the argument, we need to check whether, under these conditions, the seller would not want to encourage *other* behavioral outcomes by choosing one of the rebate options. Of these, it is easy to rule out (r1): (r1) is dominated by (r2) when $\underline{C}^h \ge \lambda\bar{C}^h$. Comparing the profits under (c3b) with the profits under (r2) and (r3), it is easy to verify that (c3b) beats the two rebate options if $\lambda(1-\alpha)(V^l - \underline{C}^l) \le \min\{\lambda(\bar{C}^l - \underline{C}^l), \lambda(\bar{C}^l - \underline{C}^l) + \alpha(\underline{C}^h - \bar{C}^l) - \lambda(1-\alpha)\bar{C}^l\}$. Otherwise, either (r2) or (r3) is optimal.

Note that when rebate is optimal via (r3), its advantage over coupon option (c3b) stems from substituting slippage for nonparticipation. While with coupons type-l consumers in the high state do not buy at all, with rebates they end up not redeeming the rebate after paying regular price up front. On the other hand, when rebate is optimal via (r2), its advantage stems from being able to substitute "good" participation—participation at a high enough promotional price—for nonparticipation.

Type-h consumers pay regular price in the high state and promotional price in the low state; type-l consumers pay the promotional price in the low state and do not purchase in the high state.

This is option (c1b) in the coupon case, and the seller cannot achieve this outcome with rebates. The conditions governing option-(c1b) optimality are $\lambda(1-\alpha)(V^l - \underline{C}^l) < (\bar{C}^l - \underline{C}^l)$ and $\underline{C}^h < \lambda\bar{C}^h$ (cf. Table 1). Under these conditions, (r1) dominates (r2). Comparing (c1b) with (r1) and (r3), we see that (c1b) beats (r1) and (r3) if $\lambda(1-\alpha)(V^l - \underline{C}^l) \le \min(\lambda(\bar{C}^l - \underline{C}^l), \lambda(\bar{C}^l - \underline{C}^l) + \alpha\lambda\bar{C}^h - (\alpha + (1-\alpha)\lambda)\bar{C}^l)$. Otherwise, either (r1) or (r3) is optimal. As before, when rebate is optimal via (r3), its advantage over (c1b) stems in part from being able to substitute slippage for nonparticipation, and when rebate is optimal via (r1), its advantage comes from being able to substitute participation for nonparticipation.

Putting all of this together, we get Table 3, which is summarized in Proposition 3 below.

Proposition 3. (1) *Rebates are better than coupons if all low-reservation-price consumers are to be served (i.e., when $\lambda(1-\alpha)(V^l - \underline{C}^l) \ge (\bar{C}^l - \underline{C}^l)$). Coupons are better than rebates only if some low-reservation-price consumers are to be excluded (i.e., only when $\lambda(1-\alpha)(V^l - \underline{C}^l) < (\bar{C}^l - \underline{C}^l)$).*

(2) *Coupons are better than rebates if and only if $\lambda(1-\alpha)(V^l - \underline{C}^l) \le \lambda(\bar{C}^l - \underline{C}^l) + \min\{0, \max(\alpha\underline{C}^h, \alpha\lambda\bar{C}^h) - (\alpha + \lambda(1-\alpha))\bar{C}^l\}$.*

Part (1) of the proposition points to the relative strengths and weaknesses of coupons and rebates as price-discrimination devices. When the objective is to serve everyone in the market while price discriminating, rebates extract more consumer surplus than coupons: Both regular and promotional prices are higher. The greater efficiency of rebates in surplus extraction comes from the bundling of probabilities and redemption-cost outcomes at the purchase stage. The consumer is forced to react to a gamble, which she does by considering *expected* redemption

Table 3 Coupons Versus Rebates

	$\lambda(1-\alpha)(V^l - \underline{C}^l) \geq (\bar{C}^l - \underline{C}^l)$	$\lambda(1-\alpha)(V^l - \underline{C}^l) < (\bar{C}^l - \underline{C}^l)$	
		$\alpha\underline{C}^h \geq (\alpha + (1-\alpha)\lambda)\bar{C}^l$	$\alpha\underline{C}^h < (\alpha + (1-\alpha)\lambda)\bar{C}^l$
$\underline{C}^h \geq \lambda\bar{C}^h$	Rebates optimal	Coupons optimal if $\lambda(1-\alpha)(V^l - \underline{C}^l) < \lambda(\bar{C}^l - \underline{C}^l)$ otherwise rebates optimal.	Coupons optimal if $\lambda(1-\alpha)(V^l - \underline{C}^l) < \lambda(\bar{C}^l - \underline{C}^l) - \Delta_1$ otherwise rebates optimal.
		$\alpha\lambda\bar{C}^h \geq (\alpha + (1-\alpha)\lambda)\bar{C}^l$	$\alpha\lambda\bar{C}^h < (\alpha + (1-\alpha)\lambda)\bar{C}^l$
$\underline{C}^h < \lambda\bar{C}^h$	Rebates optimal	Coupons optimal if $\lambda(1-\alpha)(V^l - \underline{C}^l) < \lambda(\bar{C}^l - \underline{C}^l)$ otherwise rebates optimal.	Coupons optimal if $\lambda(1-\alpha)(V^l - \underline{C}^l) < \lambda(\bar{C}^l - \underline{C}^l) - \Delta_2$ otherwise rebates optimal.

$\Delta_1 = (\alpha + (1-\alpha)\lambda)\bar{C}^l - \alpha\underline{C}^h$; $\Delta_2 = (\alpha + (1-\alpha)\lambda)\bar{C}^l - \alpha\lambda\bar{C}^h$

cost; expected redemption cost shows less variation than redemption-cost outcomes, so heterogeneity is reduced. Reduced heterogeneity is good from a price discrimination viewpoint given the limited number of tools the seller has: just regular price and promotional price. Coupons, on the other hand, by bundling buying and redemption decisions, unbundle probabilities and redemption-cost outcomes, which leads to more heterogeneity than the coupon program can handle.

This very weakness of coupons can be a strength, however. Coupons allow more fine-tuned control over who gets to buy the product. Even within consumer type l, some consumers may buy the product with a coupon, and others not buy, depending on their redemption-cost realizations. Rebates do not allow such fine control—either all consumers of type l are excluded (which, of course, means that a rebate is not needed) or the entire type buys. When the gap between type-l's reservation price and their high-state redemption cost is small—the least attractive segment is not very attractive—the seller has to pay a heavy price for this bluntness; effectively, it is forced to charge $V^l - E(C^l)$ as the promotional price, when it could be charging $V^l - \underline{C}^l$ under coupons. Coupons provide more flexibility because the seller can design the coupon such that type-l consumers in their high state do not buy.

This advantage of coupons over rebates becomes even more decisive when consumers are risk averse. Coupons provide insurance naturally: In none of the coupon options is type-l's surplus ever negative. By contrast, with rebates there is a bigger fluctuation in the consumer's fortunes, depending on which redemption-cost state is realized. In all three rebate options, type-l's consumer surplus is negative when redemption costs are high and positive when redemption costs are low. As the following corollary demonstrates, this is enough to make coupons weakly dominant.

Corollary 1. *When consumers are risk averse enough to demand nonnegative surplus in every redemption state, coupons (weakly) dominate rebates: They are better when $\lambda(1-\alpha)(V^l - \underline{C}^l) < (\bar{C}^l - \underline{C}^l)$ and equivalent otherwise.*

3.4. To Promote or Not to Promote?

The foregoing analysis was conditioned on the assumption that promotions were called for. That need not be the case. Without promotions, the seller can have an everyday high price, V^h, catering to type-h consumers only, or an everyday low price, V^l, catering to type-h and type-l consumers. In the former case, it nets αV^h, and in the latter case it nets V^l. In short, $\max(\alpha V^h, V^l)$ can be guaranteed without any promotions. This introduces another constraint into the analysis: The optimal promotion must provide a profit at least as high as this datum. The following proposition states the complete optimal promotional strategy.

Proposition 4. *When coupons are better than rebates, the seller should use coupons if $\max(0, \alpha V^h - V^l) + \lambda(1-\alpha)(V^l - \underline{C}^l) + \underline{C}^l \leq \max(\alpha\underline{C}^h, \alpha\lambda\bar{C}^h)$; otherwise, no promotion is optimal. When rebates are better than coupons, the seller should use rebates if $\max(0, \alpha V^h - V^l) + E(C^l) \leq \max(\alpha\lambda\bar{C}^h, \alpha\underline{C}^h, (\alpha + (1-\alpha)\lambda)\bar{C}^l)$; otherwise, no promotion is optimal.*

The following examples show that any of the three strategies—using rebates, using coupons, no promotion—can be optimal.

- $\lambda = 0.1$, $\alpha = 0.6$, $\bar{C}^h = 0.8$, $\underline{C}^h = 0.6$, $\bar{C}^l = 0.5$, $\underline{C}^l = 0.1$, $V^h = 2$, $V^l = 1$.

In this case, promoting with coupons is the best strategy. Segment 3—type l in the high state—is not very attractive in this example, even though segment 4—type l in the low state—is. These are exactly the conditions under which coupons thrive. Type l being attractive in the low state means that there is scope for a promotion; type l in the high state not being attractive means that some type-l consumers ought to be excluded, which only coupons can accomplish.

- $\lambda = 0.1$, $\alpha = 0.6$, $\bar{C}^h = 0.8$, $\underline{C}^h = 0.6$, $\bar{C}^l = 0.5$, $\underline{C}^l = 0.1$, $V^h = 97$, $V^l = 58$.

In this case, which has higher Vs than in the previous example, segment 3 is very profitable, which means that coupons' selective-exclusion property is not needed. Now rebates' superior ability to extract surplus comes to the fore. At the same time, given the small λ and the disparity in redemption costs, it is possible to achieve good price discrimination between the types by using a rebate—only the type-l consumers redeem—which argues for offering promotions in the first place.

- $\lambda = 0.1$, $\alpha = 0.8$, $\bar{C}^h = 0.8$, $\underline{C}^h = 0.6$, $\bar{C}^l = 0.5$, $\underline{C}^l = 0.1$, $V^h = 2$, $V^l = 1$.

The only difference between this example and the first example is that $1 - \alpha$ is smaller, making type l *as a whole* not very attractive. Now, not promoting is the best strategy.

- $\lambda = 0.8$, $\alpha = 0.6$, $\bar{C}^h = 0.8$, $\underline{C}^h = 0.6$, $\bar{C}^l = 0.5$, $\underline{C}^l = 0.1$, $V^h = 97$, $V^l = 58$.

The only difference between this example and the second example is that $1 - \lambda$, the likelihood of the low state, is much smaller. Given rebates, only type-l consumers in the low state would redeem, so not promoting is better than offering rebates.

Combined, the last two propositions answer the questions of whether to promote and how to promote. The examples illustrate the main intuitions. Coupons are the better promotion vehicle, and ought to be used, only when type-l consumers are attractive in their low state but not in their high state, making the former worth serving, but not the latter. Rebates cannot do the job under these conditions because they cannot exclude part of a consumer type. Rebates are optimal, and ought to be used, when type-l consumers in the high state are quite profitable—large $V^l - \bar{C}^l$—and it is possible to separate type-h consumers from type-l consumers at low cost. Separation is best when all type-l consumers would redeem the rebate and all type-h won't (rebate option r2). When rebate option (r1) is optimal, everyday low prices is a tempting alternative; when rebate option (r3) is optimal, everyday high prices is a tempting alternative.

The conditions for the optimality of rebates versus coupons correspond nicely to what we know about the use of these promotion vehicles in the real world (Blattberg and Neslin 1990). Rebates are primarily used for high-ticket goods like electronics, kitchen appliances, cars, etc. Coupons are primarily used for low-ticket goods like household cleaning products, foods, health and beauty aids, etc. Because redemption costs do not vary by what kind of good the promotion is attached to, the difference between high-ticket goods and low-ticket goods speaks directly to the gap $V^l - \bar{C}^l$, which is really the seller's net unit margin from selling to the *worst* segment via promotion. When this gap is substantial, it is optimal to serve these consumers via promotion, hence all consumers, and rebates are better at price-discriminating when everyone is to be served. For low-ticket goods, when this gap is small, our theory identifies coupons as the optimal solution because they provide more fine-tuned control to the seller in selectively excluding only the unattractive *part* of the type-l segment.

4. Delay Between Rebate Redemption and Rebate Payment

The model above can be extended in a number of ways. One set of extensions would test the robustness of the arguments to the technical assumptions made. For example, it would be useful to check whether qualitatively similar results hold when consumer types are continuously distributed. Other extensions may be more substantive. For example, redemption-cost states may not be independent of consumer types: High-reservation-price consumers may be more prone to high redemption-cost states than low-reservation-price consumers. For rebates, our analysis didn't consider redemption-cost state at the time of purchase because rebates are redeemed in the future, and, given our assumption of temporal independence, current redemption costs do not provide information about future redemption costs. With correlation, one would expect the difference between coupons and rebates to be attenuated. Finally, there are a number of time intervals that can be modeled, for example, the time between when a coupon/rebate is dropped and when it is redeemed, and the time between rebate redemption and when the rebate check is received by the consumer. Of these, the last is arguably the most relevant empirically, and the one that has the most potential for undermining the effectiveness of rebates vis-a-vis coupons, so we examine it here.

The time interval between when a rebate is redeemed and when the rebate check arrives, eight to ten weeks in most estimates (Greenman 1999, Edmonston 2001), is commonly seen as a particularly onerous feature of rebates from the consumer's point of view—something coupons do not have. From the seller's point of view, this time interval represents "float"—a period during which the seller has access to the money owed the (redeeming) consumer, and can make money on it by investing it. Indeed, several commentators have alluded to this as being the money-making formula for free-after-rebate vendors (see, for example, Edmonston 2001).

Let ρ_s and ρ_c denote the discount factors that apply to this interval from the seller's and consumers' points of view, respectively. Note that ρ_s captures within it the benefit the seller realizes from investing the amount owed the redeeming consumer: The lower ρ_s is, the lower the real cost of the rebate to the seller. From the consumer's viewpoint, the lower ρ_c is, the lower the value of the rebate. In fact, it is possible that ρ_c is so small that the rebate is not worth redeeming.

Clearly, the introduction of a time delay between rebate redemption and rebate-check arrival does not affect the coupon analysis in any way. Therefore, consider the rebate options—in particular, option (r1). The seller's problem can now be written as

$$\begin{aligned} \max_{p,\,p_l}\ & p-\rho_s(1-\alpha\lambda)(p-p_l), \\ \text{s.t. }\ & V^l-p-[E(C^l)-\rho_c(p-p_l)]\geq 0, \\ & V^h-p-[(1-\lambda)(\underline{C}^h-\rho_c(p-p_l)]\geq 0, \\ & \underline{C}^h\leq \rho_c(p-p_l)\leq \bar{C}^h. \end{aligned}$$

Note that in each of the constraints only the rebate is being discounted, not the redemption cost. This is because we have implicitly assumed that the time between purchase and rebate redemption is insignificant. Once again, $V^h-E(C^h)\geq V^l-E(C^l)$ implies that only the first and third constraints are binding, and the latter in the upper bound. Therefore, $p=V^l-E(C^l)+\bar{C}^h$ and $p_l=p-(\bar{C}^h/\rho_c)$, and the seller makes $V^l-E(C^l)+[1-(\rho_s/\rho_c)(1-\alpha\lambda)]\bar{C}^h$ in expected profit.

Similarly, under option (r2), the seller now sets $p_l=p-(\underline{C}^h/\rho_c)$ and $p=V^l+\underline{C}^h-E(C^l)$, which yields $V^l-E(C^l)+[1-(\rho_s/\rho_c)(1-\alpha)]\underline{C}^h$; and under option (r3), the seller sets $p_l=p-(\bar{C}^l/\rho_c)$ and $p=V^l+\bar{C}^l-E(C^l)$, which yields $V^l-E(C^l)+[1-(\rho_s/\rho_c)(1-\alpha)(1-\lambda)]\bar{C}^l$.

Note that the regular price is unaffected by the introduction of discount factors, but the promotional price is lower—the seller has to compensate type-l consumers for the fact that it takes some time to get the rebate check. But, of course, the seller holds on to this money longer and potentially earns interest on it—all of which is reflected in ρ_s—so the question remains whether in the end the seller is better off or worse off. Comparing profits, we see that there is a difference if and only if the seller's discount factor is not the same as the consumers'. In other words, the time delay between rebate redemption and rebate payment does not automatically make a difference to the seller's profit. If seller and consumers are equally patient, whatever the seller has to pay in compensation to the consumers in the form of lower promotional prices, it makes up on the "front end" by holding onto, and investing, the money owed. As might be guessed, if the consumers are more (less) impatient than the seller, then the seller's profit is lower (higher) in present-value terms relative to the benchmark case of no time delay.

How does the introduction of a time delay between rebate redemption and rebate arrival affect the comparison between coupons and rebates? Consider coupon (c1a)—the relevant comparison for rebate (r1). Because the regular price under rebates is unaffected by discount factors, it continues to be higher than the regular price under coupons. However, the promotional price under rebates might be lower now if the consumers' discount factor is very small. Simple calculations show that under equal discount factors for the seller and the consumers, the profit under rebate continues to be higher, but now the possibility arises that if the consumers' discount factor is much lower than the seller's, then coupons might be better than rebates.

Proposition 5. *The delay between rebate redemption and rebate payment does not change Proposition 4 as long as the seller's discount factor is the same as the consumer's. However, when $\rho_s>\rho_c$, the delay between rebate redemption and rebate payment narrows the area of dominance of rebates over coupons, and vice versa when $\rho_s<\rho_c$.*

Of course, to some extent, the delay between rebate redemption and rebate payment is under the seller's control. Arguably, if the seller's profit was going down because of the delay, then the seller would try to shorten it—if it was possible to do so cost effectively.

5. Conclusion

We have compared coupons and rebates in terms of how they work as price-discrimination devices, recognizing the definitional difference between them—namely, that coupons are redeemed with the purchase, while rebates are redeemed after the purchase. When consumers face uncertainty about redemption costs, we show that this difference implies different advantages and disadvantages for the two promotion vehicles. By unbundling the redemption decision from the purchase decision, rebates present a bundled offer to the consumer: They can buy or not buy the product. Coupons, on the other hand, bundle the redemption decision with the purchase decision, but in the process they present an unbundled offer: The consumer can buy or not buy the product, with or without the coupon. Rebates, therefore, are more efficient at surplus extraction, whereas coupons offer more fine-tuned control over who buys the product. We show that the resulting trade-offs are resolved by looking at the gap between low-reservation-price consumers' valuation of the product and their highest redemption cost. When the gap is large (relative to the fluctuation in redemption costs), as is likely the case for high-ticket goods, rebates are indicated; otherwise, as is likely the case for low-ticket goods, coupons are indicated. Risk aversion on the part of consumers reduces the attractiveness of rebates, as does the time delay between rebate redemption and rebate payment, but the latter if and only if consumers are more impatient than the seller. These predictions are consistent with real-world practice.

Much has been written about the fact that many consumers do not redeem either promotion vehicle, but whereas with coupons this is generally seen as a negative, with rebates it is generally seen as a positive. This is presumably because, with rebates, the consumer has already paid regular price when the redemption question comes up. Practitioners view this "slippage advantage" of rebates over coupons as a big advantage, but our analysis suggests that rational consumers will take the risk of nonredemption into account when evaluating rebates, and penalize them appropriately. In fact, the penalty might be so high that coupons dominate rebates when consumers are risk averse. In general, our analysis reveals the folly of focusing on redemption numbers as a managerial objective. Redemption numbers are the outcomes of promotion design, not an end in itself. If the goal is profit maximization, the right level of redemption depends on the parameters of the promotion environment. For coupons and rebates, "low," "intermediate," and "high" levels of redemption may all be optimal under the right circumstances. What those circumstances are differs between coupons and rebates. In equilibrium, rebate redemptions and rebate value covary, as a function of redemption costs, segment sizes, and the likelihood of redemption-cost states; product value does not enter the picture. With coupons, however, equilibrium coupon value is independent of product value, but redemptions are not. As a result, the relationship between coupon value and redemptions is looser: The same coupon value may lead to different levels of redemptions depending on product value.

We would like to close by emphasizing the need to think about coupons and rebates in a slightly more imaginative way than what current industry practice would have us do. Coupons are not just FSI coupons, and rebates are not just mail-in rebates; nor are coupons limited to frequently purchased low-ticket goods and rebates to high-ticket durable goods. The model offered here, as encapsulated in Figure 1, forces a broader interpretation. The creativity comes in how we interpret redemption costs and how we interpret promotional discounts. What is immutable are the basics of promoting for price discrimination: The promotion vehicle must impose redemption costs on which consumers differ; it must offer rewards for people willing to bear the redemption cost; and coupons differ from rebates in the timing of the reward. Seen in this light, a coupon on the shelf, in a *grocery* store, is like a price cut (although not exactly so; see Dhar and Hoch 1996). Its price discrimination ability is low because the consumer's redemption costs are low: The weekly trip to the grocery store is preordained and it does not cost that much to pick up a coupon sitting on a store shelf (Banks and Moorthy 1999). By contrast, for a durable goods retailer, a temporary price cut is like a price-discriminating coupon; the special trip to the store before the sale expires is the redemption cost, and if a consumer is willing to bear it, a lower price is her reward. Similarly, reward programs like airline frequent-flyer programs (e.g., Kim et al. 2001) are, in fact, rebate programs (with large redemption costs; see Lieber and Brannigan 2002). And so, are, for that matter, the in-pack and on-pack—but not instantly redeemable—coupons found on consumer packaged goods (Raju et al. 1994).

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Appendix

In the main case analyzed in the text we assumed risk neutrality, no-rebate-payment delay, and $\bar{C}^h > \underline{C}^h > \bar{C}^l > \underline{C}^l$, $V^h - V^l \geq \bar{C}^h - \underline{C}^l$. Here we verify that changing the cost and value-ordering assumptions to: (1) $\bar{C}^h > \bar{C}^l > \underline{C}^h > \underline{C}^l$ and $V^h - V^l \geq \bar{C}^h - \underline{C}^l$, (2) $\bar{C}^h > \underline{C}^h > \bar{C}^l > \underline{C}^l$ and $V^h - V^l < \bar{C}^h - \underline{C}^l$, and (3) $\bar{C}^h > \bar{C}^l > \underline{C}^h > \underline{C}^l$ and $V^h - V^l < \bar{C}^h - \underline{C}^l$, will not change our results substantively.

1. $\bar{C}^h > \bar{C}^l > \underline{C}^h > \underline{C}^l$, $V^h - V^l \geq \bar{C}^h - \underline{C}^l$

With respect to coupons, the key difference from before is that option (c2), in which type-h consumers pay regular price and type-l consumers pay promotional price, is infeasible. Because high-state consumers of type l have a bigger redemption cost than low-state consumers of type h, if high-state consumers of type l use coupons, so will low-state consumers of type h. The impact of this change is that while $\lambda(1-\alpha)(V^l - \underline{C}^l) \geq (\bar{C}^l - \underline{C}^l)$ still determines whether or not option (c1a) beats option (c1b), it is possible that option (c1b) is beaten by option (c3b) because $\underline{C}^h \geq \lambda\bar{C}^h$. In that case, the choice between option (c1a) and (c3b) revolves around whether $\lambda(1-\alpha)(V^l - \underline{C}^l) \geq (\bar{C}^l - \underline{C}^l) + \alpha(\underline{C}^h - \lambda\bar{C}^h)$ or not. In other words, the scope for options (c1a) and (c3b) to be best has increased; now it is possible for option (c1a) to be best even when $\underline{C}^h \geq \lambda\bar{C}^h$ provided $\lambda(1-\alpha)(V^l - \underline{C}^l) \geq (\bar{C}^l - \underline{C}^l) + \alpha(\underline{C}^h - \lambda\bar{C}^h)$ and, similarly, it is possible now for option (c3b) to be best even when $\lambda(1-\alpha)(V^l - \underline{C}^l) \geq (\bar{C}^l - \underline{C}^l)$ provided $\underline{C}^h \geq \lambda\bar{C}^h$ and $\lambda(1-\alpha)(V^l - \underline{C}^l) < (\bar{C}^l - \underline{C}^l) + \alpha(\underline{C}^h - \lambda\bar{C}^h)$.

With respect to rebates, also, option (r2) is not feasible because of the overlapping cost structure, but now the seller has a new option: All consumers redeem the rebates in the low state, but not in the high state. However, this option implies $p_l = V^l - E(C^l)$, $p = p_l + \bar{C}^l$ and profits of

Table 4 Coupon Options

c1a	c1b	c2	c3b
Profit function $\alpha(\lambda p+(1-\lambda)p_l)+(1-\alpha)p_l$	Profit function $\alpha(\lambda p+(1-\lambda)p_l)+(1-\alpha)(1-\lambda)p_l$	Profit function $\alpha p+(1-\alpha)p_l$	Profit function $\alpha p+(1-\lambda)(1-\alpha)p_l$
Constraints $V^h-p\geq 0$, $V^h-p_l\geq \underline{C}^h$, $V^l-p_l\geq \bar{C}^l$, $\bar{C}^h\geq p-p_l\geq \underline{C}^h$.	Constraints $V^h-p\geq 0$, $V^h-p_l\geq \underline{C}^h$, $\bar{C}^l>V^l-p_l\geq \underline{C}^l$, $\bar{C}^h\geq p-p_l\geq \underline{C}^h$.	Constraints $V^h-p\geq 0$, $V^l-p_l\geq \bar{C}^l$, $\underline{C}^h\geq p-p_l\geq \bar{C}^l$.	Constraints $V^h-p\geq 0$, $\bar{C}^l>V^l-p_l\geq \underline{C}^l$, $\underline{C}^h\geq p-p_l\geq \underline{C}^l$.
Solution $p_l=V^l-\bar{C}^l$, $p=p_l+\bar{C}^h$ $\pi=V^l-\bar{C}^l+\alpha\lambda\bar{C}^h$.	Solution $p_l=V^l-\underline{C}^l$, $p=p_l+\bar{C}^h$ $\pi=(V^l-\underline{C}^l)[1-\lambda(1-\alpha)]+\alpha\lambda\bar{C}^h$.	Solution $p_l=V^l-\bar{C}^l$, $p=p_l+\underline{C}^h$ $\pi=V^l-\bar{C}^l+\alpha\underline{C}^h$.	Solution $p_l=V^l-\underline{C}^l$, $p=p_l+\underline{C}^h$ $\pi=(V^l-\underline{C}^l)[1-\lambda(1-\alpha)]+\alpha\underline{C}^h$.

$V^l-(1-\lambda)\underline{C}^l$, which is worse than not offering rebates at all. So, the seller really has only two options to choose from: options (r1) and (r3). Option (r1) is exactly the same as before, and option (r3) changes in only one respect: The constraint $\underline{C}^l\leq p-p_l\leq \bar{C}^l$ gets modified to $\underline{C}^l\leq p-p_l\leq \underline{C}^h$. This implies that now $p_l=V^l-\lambda\underline{C}^h-(1-\lambda)\underline{C}^l$ and $p=p_l+\underline{C}^h$, with profits $V^l+\alpha(1-\lambda)\underline{C}^h-(1-\lambda)\underline{C}^l$. By comparing the profits of options (r1) and (r3), we see that if $\alpha\lambda\bar{C}^h\geq \lambda\bar{C}^l+\alpha(1-\lambda)\underline{C}^h$, option (r1) should be chosen, otherwise, option (r3) should be chosen.

In choosing between coupons and rebates, because options (c1a) and (r1) are unchanged, rebates continue to be better than coupons if the goal is to get type-h consumers in the high state paying regular price and all others paying the promotional price. On the other hand, if the goal is to have type-h consumers paying regular price and type-l consumers paying the promotional price in the low state and not purchasing in the high state, then only coupon option (c3b) can achieve this. The conditions governing option-(c3b) optimality are slightly different from before, namely $\lambda(1-\alpha)(V^l-\underline{C}^l)<(\bar{C}^l-\underline{C}^l)+\alpha(\underline{C}^h-\lambda\bar{C}^h)$ and $\underline{C}^h>\lambda\bar{C}^h$. Of course, it is still possible that when option (c3b) is the best couponing strategy, a rebate achieving (r1) or (r3) beats it. However, rebate option (r1) is dominated by rebate option (r3) under $\underline{C}^h>\lambda\bar{C}^h$ (because $\underline{C}^h>\lambda\bar{C}^h$ guarantees $\alpha\lambda\bar{C}^h<\lambda\bar{C}^l+\alpha(1-\lambda)\underline{C}^h$), under which option (r3) is better than option (r1). Comparing the profits under (c3b) with the profits under (r3), it is easy to verify that (c3b) beats the rebate if $\lambda(1-\alpha)(V^l-\underline{C}^l)\leq \lambda(\alpha\underline{C}^h-\underline{C}^l)$. Finally, consider the outcome: Type-h consumers pay regular price in the high state and promotional price in the low state; type-l consumers pay the promotional price in the low state and do not purchase in the high state. This is option (c1b) in the coupon case, and the seller cannot achieve this outcome with rebates. So, as before, *if* achieving this outcome is optimal, then coupons are needed in the form of (c1b). Option (c1b) is the optimal couponing strategy when $\lambda(1-\alpha)(V^l-\underline{C}^l)<(\bar{C}^l-\underline{C}^l)$ and $\underline{C}^h<\lambda\bar{C}^h$. Comparing option (c1b) with options (r1) and (r3), we see that (c1b) is better than (r1) if and only if $\lambda(1-\alpha)(V^l-\underline{C}^l)\leq \lambda(\bar{C}^l-\underline{C}^l)$. Comparing option (c1b) with option (r3), (c1b) is better than (r3) if and only if $\lambda(1-\alpha)(V^l-\underline{C}^l)\leq \alpha(\lambda\bar{C}^h-(1-\lambda)\underline{C}^h)-\lambda\underline{C}^l$. In other words, (c1b) beats (r1) and (r3) if $\lambda(1-\alpha)(V^l-\underline{C}^l)\leq \min(\lambda(\bar{C}^l-\underline{C}^l),\alpha(\lambda\bar{C}^h-(1-\lambda)\underline{C}^h)-\lambda\underline{C}^l)$. Combining all of the above, we get the following decision criterion: If $\lambda(1-\alpha)(V^l-\underline{C}^l)\leq -\lambda\underline{C}^l+\min(\lambda\bar{C}^l,\max(\alpha\lambda\bar{C}^h-\alpha(1-\lambda)\underline{C}^h,\lambda\alpha\underline{C}^h))$, rebates are better than coupons; otherwise, coupons are better. This condition is qualitatively similar to the condition in Proposition 3.

2. $\bar{C}^h>\underline{C}^h>\bar{C}^l>\underline{C}^l$, $V^h-V^l<\bar{C}^h-\underline{C}^l$

As far as coupons are concerned, options (c1a), (c2), and (c3b) are still the same as the respective parts in the main case, and only option (c1b) changes. Here, too, the constraints are still the same as in the main case, but the

Table 5 Rebate Options

r1	r2	r3
Profit function $\alpha(\lambda p+(1-\lambda)p_l)+(1-\alpha)p_l$	Profit function $\alpha p+(1-\alpha)p_l$	Profit function $\alpha p+(1-\alpha)((1-\lambda)p_l+\lambda p)$
Constraints $V^h-p+(1-\lambda)[(p-p_l)-\underline{C}^h]\geq 0$ $V^l-p_l\geq E(C^l)$, $\bar{C}^h\geq p-p_l\geq \underline{C}^h$.	Constraints $V^h-p\geq 0$, $V^l-p_l\geq E(C^l)$, $\underline{C}^h\geq p-p_l\geq \bar{C}^l$.	Constraints $V^h-p\geq 0$, $V^l-p+(1-\lambda)[(p-p_l)-\underline{C}^l]\geq 0$ $\bar{C}^l\geq p-p_l\geq \underline{C}^l$.
Solution $p_l=V^l-E(C^l)$, $p=p_l+\bar{C}^h$ $\pi=V^l-E(C^l)+\alpha\lambda\bar{C}^h$.	Solution $p_l=V^l-E(C^l)$, $p=p_l+\underline{C}^h$ $\pi=V^l-E(C^l)+\alpha\underline{C}^h$.	Solution $p_l=V^l-E(C^l)$, $p=p_l+\bar{C}^l$ $\pi=V^l-E(C^l)+(\alpha+(1-\alpha)\lambda)\bar{C}^l$.

difference is that now V^h and V^l are so close that $p_l = V^l - \underline{C}^l$, $p = V^h$, and the seller's profit is $\alpha\lambda V^h + (1-\lambda)(V^l - \underline{C}^l)$. For rebates, all three alternatives are the same as in the main case. Comparing coupons with rebates, we get: When $\lambda(1-\alpha)(V^l - \underline{C}^l)$ is bigger than $\max\{\alpha(\underline{C}^h - \lambda\underline{C}^l), \alpha\lambda(V^h - V^l)\} + \min\{-\alpha(1-\lambda)\bar{C}^l, \lambda\bar{C}^l - \alpha\underline{C}^h, \lambda(\bar{C}^l - \alpha\bar{C}^h)\} - \lambda(1-\alpha)\underline{C}^l$, rebates are better than coupons; otherwise, coupons are better than rebates.

3. $\bar{C}^h > \bar{C}^l > \underline{C}^h > \underline{C}^l$, $V^h - V^l < \bar{C}^h - \underline{C}^l$

This case behaves like a "convex" combination of the previous two extensions. Option (c2) is infeasible, and option (c1b) changes from the main case as above. For rebates, option (r2) is infeasible; (r1) and (r3) are the same as in the main case. Comparing coupons with rebates, we get: When $\lambda(1-\alpha)(V^l - \underline{C}^l)$ is bigger than $\max\{\alpha(\underline{C}^h - \lambda\underline{C}^l), \alpha\lambda(V^h - V^l)\} + \min\{-\alpha(1-\lambda)\underline{C}^h, \lambda(\bar{C}^l - \alpha\bar{C}^h)\} - \lambda(1-\alpha)\underline{C}^l$, rebates are better than coupons; otherwise, coupons are better than rebates.

In short, variations in the cost and value structure that preserve the basic inequalities $E(C^h) > E(C^l)$, $V^h > \bar{C}^h$, and $V^l > \bar{C}^l$ do not change our main results substantively.

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