Discovering Optimal Release Conditions for the Javelin

World Record Holders by Using

Computer Simulation

Ching-Hua Chiu*

Graduate Institute of Sports & Health Management, National Chung Hsing University, Taichung, Taiwan, 402, ROC

Abstract

The purpose of this study is to use computer simulation to establish the optimal release conditions for javelin world record holders. The researcher adopted the data on the relationship between the attack angle and the distance from COM (the javelin’s center of mass) to COP (the javelin's center of pressure), and used javelins which complied with the new rules [1]. Then numerical methods were used to estimate the ranges of the javelins throw at zero wind speed and other wind speeds. It was found that when the wind speed and the angular velocity were both zero and the grip height at release was 1.88m, the male world record holder could reach the world record of 94.48m by throwing the javelin of the Apollo Olympic New Rules (AONR) under the optimal release conditions—the release angle being 42.8°, the attitude angle being 35.5°, the angle of attack being -7.3°, and the release speed being 31.213m/s. However, when the male holder used the javelin of the Held New Rules (HNR), the optimal release conditions would comprise the release angle of 37.3°, the attitude angle of 31.8°, the angle of attack of -5.5°, and the release speed of 29.935m/s. When the wind speed and angular velocity were both zero and the grip height at release was 1.66m, the female world record holder could reach the world record of 72.28m using the javelin of the Apollo Ladies (AL) under the optimal release conditions—the release angle being 42.6°, the attitude angle being 34.6°, the angle of attack being -8.00°, and the release speed being 26.672m/s. Second, when the wind speed ranged from -6m/s to 6m/s, the optimal range would occur at the wind speed of 2m/s for the female holder using the AL javelin. Therefore, it could be concluded that a tail wind would help the male holder using the AONR javelin and the female holder using the AL javelin to reach their optimal performance. On the other hand, a head wind would help the male holder using the HNR javelin to achieve the optimal performance. Third, the simulation results also showed that when the male holder threw the javelin at the release speed ranging from 29m/s to 33m/s, there would be a margin of 7.5% to 10.2% in the range between AONR javelin and HNR javelin, a margin of 8.6% to 13.3% in the optimal release angle. Fourth, the distance from COM to COP was one of the factors that influenced the release angle. It was found that when the COM was farther away from the COP, the release angle would be larger and the throwing range would be shorter.

Keywords: Release angle, Attack angle, Attitude angle

Introduction

How to help javelin throwers achieve better performance is the concern of the researcher in this study. To attain better performance, one of the requirements is to throw the javelin under optimal release conditions. Therefore, finding the optimal release parameters is the issue this study attempts to deal with.

Basically, two factors are involved in determining the range of the javelin throw, one of which is the javelin’s structure and the other is the release conditions. The javelin’s structure consists of the moment of inertia relative to COM, the surface area, the lateral drag area, the skin friction drag coefficient, the lateral pressure drag coefficient and the influence of the distance from COP to COM on the attack angle [1-2]. The release conditions involve grip height at release, the release speed, the release angle, the attack angle, the attitude angle, the angular velocity and the wind speed the competitors encounter [1-2]. The old-fashioned javelins refer to those produced before 1986. The aerodynamic rules were widely applied in producing them. As a result, the efficiency of the javelin throw was improved. In 1984, Uwe Hohn established the world record of 104.80m. This amazing distance jeopardized the other competitors and referees [1,3-4], forcing the International Amateur Athletics Federation to set new rules for making javelins (NRJ) in 1986. With NRJ, the structure of javelin was altered in two aspects: the mass of the new-style javelin was moved 4cm toward the tip; the maximal diameter for the tail of the new-style javelin was restricted [1,5]. Due to the two alterations, the resulting distance was shortened by 12 to 13% [1].

As for the release conditions for the javelin throw, most researchers in the past focused their attention on the kinematics and the analysis of release parameters [5-8], or the wind tunnel...
test (WT T) for the drag force and lift force in javelin throw [9].

Though there are some previous studies working on the optimal release conditions, but the adopted numerical methods, the hypothesized physics terms, and the brands of javelins were all different; as a result, these studies presented different optimal conditions. For example, Soong [2] once hypothesized that in the flight of the old-fashioned javelin, the distance from COM to COP was fixed. With this hypothesis, he estimated the optimal release angle. However, Soong’s assumption didn’t correspond to the finding in WTT. The studies conducted by Terauds with WTT in 1972 and 1985 [1] demonstrated that whether it was new-style or old-fashioned javelin, the distance from COM to COP was not fixed, but varied with the change of the attack angle. On the other hand, Best et al. [9] estimated the optimal release angle, attack angle and angular velocity for javelins of different brands. They found that the optimal release angle was not a fixed value, but became larger with the increase of the release speed.

It can be learned from the above-mentioned literature that different structures of javelin will lead to different optimal release conditions. Consequently, to establish the optimal release conditions and thus to help contestants get better performance, the javelin structure of different brands must be taken into consideration. In view of this, the study adopted the data provided by Terauds in 1985 on the relationship between the distance from COM to COP and the change of the attack angle [1]. Then the numerical methods were used to estimate the optimal javelin release conditions for the male and female world record holders.

### Nomenclature

- $g$: acceleration of gravity, $9.81 \text{m/s}^2$
- $V_s$: release speed
- $R$: range (distance throw)
- $m$: mass
- $I$: moment of inertia
- $\rho$: air density
- $G$: gravity vector ($=[0,0,-g]^{T}$)
- $\alpha$: horizontal angle
- $\beta$: release angle
- $V_0$: release velocity vector
- $P_0$: release position vector
- $\varphi_o$: initial attack angle
- $\delta_o$: initial attitude angle
- $T_0$: initial relative air velocity vector
- $\delta_l$: landing angle
- $H_{max}$: the maximal flight height
- $\lambda$: wind speed
- $W$: wind velocity vector
- $r$: Distance from the tip of the javelin to COM
- $d$: distance from the COM to the COP of the javelin
- $Q$: landing position vector
- $\phi$: flying time
- $\Delta$: a unit time interval
- $t_k$: the position of the flying javelin in $k$ time point ($k=0,1,2,\ldots,n$)
- $\varphi_k$: the attack angle in the time point $t_k$
- $F_k$: the force on javelin in the time point $t_k$
- $P_k$: the COM vector in the time point $t_k$
- $V_k$: the COM velocity vector in the time point $t_k$
- $A_k$: the COM acceleration vector in the time point $t_k$
- $t_k$: the relative air velocity vector in the time point $t_k$
- $T_k$: the relative air speed in the time point $t_k$
- $U_k$: the unit vector of the lift force
- $U_y$: the unit vector on $y$-axis of the coordinate system oxyz
- $UD_k$: the unit vector of the drag force in the time point $t_k$
- $M_k$: The pitching moment on $y$-axis in the time point $t_k$
- $||a||$: Magnitude of $a$
- $|a|$: absolute value
- $\delta_o$, $\delta_l$, $\delta$ : the attitude angle, the angular velocity, the angular acceleration
- $C_{pk}$: the distance from the COM to the COP of the javelin in the time point $t_k$

### Materials and Methods

According to the rules of IAAF, the minimal weight of the javelin for male competitors is 0.805kg and the moment of inertia is 0.423kg m²; the minimal weight of the javelin for female competitors is 0.618kg and the moment of inertia is 0.223kg m² [2]. The javelins used for simulation in this study are all in accordance with the rules. The surface area $S_f$ for the male javelin is 0.16016 m², the lateral drag area $S_p$ is 0.04738 m² [2], and $r$, the distance from the tip to the COM of the javelin, is 1.039m. The surface area $S_f$ for the female javelin is 0.14391 m², the lateral drag area $S_p$ is 0.04581 m², and $r$, the distance from the tip to the COM of the javelin, is 0.933m.

As Fig. 1c demonstrated, the Cartesian coordinate system OXYZ was presumed to be a fixed coordinate system. In the Cartesian coordinate system OXYZ, $P_0(x_o,y_o,z_o)$ referred to the initial release position vector. $Z_o$, represented the grip height at release, $X_o$, $Y_o$ represented the positions for plane x and plane y in the Cartesian coordinate system. Before using the computer to estimate the throwing range, the researcher had to key in release parameters, including $\lambda$, the wind speed; $Z_o$, the grip height at release; $Vs$, the release speed; $\beta$, the release angle; $\varphi_o$, the attack angle; $\delta_o$, the attitude angle, and $\delta_l$, the angular velocity. Besides, for the convenience of calculation, the following three parameters were assumed to be zero: the horizontal angle $\alpha$ and the horizontal release position $X_o = \theta$, $Y_o = \theta$.

Fig. 1c also showed a moving coordinate system OXYZ’, which was established from the Cartesian coordinate system OXYZ. $P_0$ represented the origin of the moving coordinate system OXYZ’, and also was the initial release position of the javelin. Besides, another new coordinate system OXYZ was established. It was created when it rotated $\alpha$ angle along the Z’-axis of the moving coordinate system OXYZ’, and then rotated $\beta$ angle along the y-axis of the new coordinate system. $V_0$ represented the velocity position at release, which ran parallel with the x-axis of the new coordinate system oxyz. $\beta$, the release angle; $\delta_o$, the attitude angle.
Figure 1. (a) and (b) : the javelin's attack angle and moment formed due to the relative velocity vector $kT$ (c) : the coordinate systems and the related parameters at release.

$W$ referred to the velocity vector; $T_n$, the relative air velocity vector at release (Fig. 1c). When the position of COM was higher than that of COP, the attack angle $\phi$ would be defined as positive, or negative otherwise (Fig. 1a-b).

This study simulated the javelin's flying process, applied the numerical methods to estimate the range, and found the optimal range (Eq.1-3) [11]. First, in the flying process of the javelin from release to landing, $t_k$ ($k = 0, 1, ..., n$) represented a certain time point of that flight. Figure 1 demonstrated the fixed coordinate system $OXYZ$, in which the position vector of COM in $t_k$ represented by $P_k = [X_k, Y_k, Z_k]^T$. When two neighboring time points $t_k$ and $t_{k+1}$ of the javelin flight were selected, and $t_k$ was made to be very close to $s_k$, then the interval of these two time points, $\Delta t = t_{k+1} - t_k$, would be tiny. When Soong [2] estimated the throwing range of the javelin, the value of $\Delta t$ was assumed to be 0.01sec. The resulting relative error for his estimation was very tiny. Therefore, this study also set $\Delta t$ to be 0.01sec as Soong did.

When $\Delta t$ was 0.01 sec, the movement of the COM from $t_k$ to $t_{k+1}$ could be considered to be kinematics for constant acceleration [2]. Then, $P_{k+1}$, the position vector in $t_{k+1}$, and $V_{k+1}$, the velocity vector in $t_{k+1}$, could be written as followed:

$$P_{k+1} = P_k + V_k \Delta t + \frac{1}{2} A_k \Delta t^2$$  \hspace{1cm} (1)

$$V_{k+1} = V_k + A_k \Delta t$$  \hspace{1cm} (2)

$V_k$ represented the velocity vector of the javelin's COM and $A_k$ stood for the acceleration vector in the time point $t_k$ (Eq.1-2). In the equation, $A_k = F_k/m$ and $m$ meant the mass of the javelin. $F_k$ referred to the force the javelin took on during the flight, including the gravity, aerodynamic drag force and lift force; therefore, $F_k$ in the time point $t_k$ could be written as followed:

$$F_k = D_k |UD_k| + L_k |UL_k| + mG$$  \hspace{1cm} (3)
The symbol $\mathbf{G}$ represented the acceleration vector of mass and $\mathbf{G} = [0, 0, g]^T$. $g$ was the acceleration of gravity $9.81 \text{m/s}^2$. The equations for the drag force and lift force imposed on the flying javelin were presented in Appendix A, so $D_k$ and $L_k$ (Eq.3) could be written as followed:

$$D_k = \frac{1}{2} \rho v_k^2 (C_d p S_p \sin^3 \Omega_k - C_l f \sin \Omega_k \cos \Omega_k)$$

$$L_k = \frac{1}{2} \rho v_k^2 (C_d p S_p \sin \Omega_k \cos \Omega_k \sin \Omega_k \cos \Omega_k)$$

$S_p$ represented the lateral drag area; $S_f$ the surface area; $C_d$ the skin friction drag coefficient; $C_l f$ the lateral pressure drag coefficient. When $t_k = 0$, the initial release position vector $P_0 = [X_o, Y_o, Z_o]^T$. $V_o$ was defined as the release velocity vector. Since the COM of the javelin flew along the x-axis of the new coordinate system oxyz (Fig. 1c), $V_o$ could be written as followed:

$$V_o = A_3 \langle \alpha, \beta \rangle [V_x, 0, 0]^T \quad (6)$$

$$A_3 \langle \alpha, \beta \rangle = \begin{bmatrix} \cos \alpha \cos \beta & -\sin \alpha & -\cos \alpha \sin \beta \\ \sin \alpha \cos \beta & \cos \alpha & -\sin \alpha \sin \beta \\ \sin \beta & 0 & \cos \beta \end{bmatrix}$$

In equations 6 and 7, $V_o$ represented the release speed, and $A_3 \langle \alpha, \beta \rangle$ stood for the coordinate transformation matrix (Danielson, 1992). $T_o$ represented the initial relative air velocity vector (Fig. 1c) could be written as followed:

$$T_o = W - V_o \quad W = [L \cos \alpha, L \sin \alpha]^T$$

If the relative air velocity vector in the time point $t_k$ was written as $T_k = [T_{x_k}, T_{y_k}, T_{z_k}]^T$, then the attack angle $\phi_k$ in $t_k$ could be obtained by incorporating the velocity vector elements of $T_k$:

$$v_k = T_{x_k} / (T_{x_k}^2 + T_{y_k}^2)^{1/2}$$

$$\phi_k = \delta_k - \frac{\pi}{2} - \sin^{-1} v_k$$

In the new coordinate system oxyz, $U_y$ , the unit vector of y-axis, could be elaborated with the following equation:

$$U_y = A_3 \langle \alpha, \beta \rangle [0, 1, 0]^T \quad (11)$$

Providing that there were three different attack angles, then the unit vector of the lift force $U L_k$ (Eq.3) could be different accordingly. The three equations for $U L_k$ could be elaborated as followed:

$$\pi / 2; \phi_k \neq 0 \quad U L_k = U_y \times V_k = \|U_y \times V_k\|$$

$$-\pi / 2; \phi_k \neq 0 \quad U L_k = V_k \times U_y = \|V_k \times U_y\|$$

$$\phi_k = 0 \quad \text{the lift force was zero}$$

In the time point $t_k$ , $\Omega_k = \pi - \phi_k$ (Eq.5-6). $\phi_k$ represented the attack angle in $t_k$ during the flight of javelin (Fig. 1a-b). In Fig. 1b, the attack angle $\phi_k$ was defined as negative when the lift force was upward. In Fig. 1a, $\phi_k$ was defined as positive when the lift force was downward. Since COP and COM were at different positions, the COP, influenced by the air drag and lift forces, would rotate in the flying process. The consequent pitching moment $M_k$ relative to COM could be written as followed:

$$\frac{\pi}{2}; \phi_k \neq 0 \quad M_k = -(C_p_k \sin \phi_k D_k + C_l_p_k \cos \phi_k L_k)$$

$$-\frac{\pi}{2}; \phi_k \neq 0 \quad M_k = (C_p_k \sin \phi_k D_k + C_l_p_k \cos \phi_k L_k)$$

$$\phi_k = 0 \quad M_k = 0$$

In this study, the researcher decided to ignore the rolling of the horizontal axis and the rotation of the vertical axis, and then the angular acceleration of y-axis could be written as $\delta_k = M_k \Omega_k + V_o$. So the attack angle position $\delta_k + \delta_k$ and angular velocity in the time point $t_k$ could be written as followed:

$$\delta_k + 1 = \delta_k + \delta_k + \frac{1}{2} \delta_k \Delta t^2$$

$$\delta_k + 1 = \delta_k + \delta_k$$

Supposing the position vector of COM in the landing time $t_n$ was $P_n$ and $P_n = [X_n, Y_n, Z_n]^T$ (Fig.2), then the position vector in the javelin landing would be presented as $Q = [Q_x, Q_y, Q_z]^T$. Its equation was as followed:

$$Q = P_n + A_3 \langle \alpha, \beta \rangle [r \cos \theta, \sin \theta]^T \quad (16)$$

$$A_3 \langle \alpha, \beta \rangle = \begin{bmatrix} \cos \alpha \cos \beta & -\sin \alpha & -\cos \alpha \sin \beta \\ \sin \alpha \cos \beta & \cos \alpha & -\sin \alpha \sin \beta \\ \sin \beta & 0 & \cos \beta \end{bmatrix}$$

The straight throwing range of the javelin:

$$R = (Q_x - X_0)^2 + (Q_y - Y_0)^2 + Z_n^2$$

As for $d$ , the distance from COP to COM of the javelin, this study referred to the data provided by Terauds (1985) on the relationship between the attack angle and the distance $d$ [1]. Several data points were selected for the curve fitting for $d$ , and the spline Interpolation was used to conduct spline curve fitting on these selected data points [14]. Then the value
of $C_{pk}$ at the attack angle $\phi_k$ could be calculated. In 1985 Terauds presented the data on the relationship between the attack angle and the distance from COM to COP [1,9]. His data comprised three parts—the male using the AONR javelin, the male using the HNR javelin, and the female using the AL javelin [1].

Results and Discussion

Under the normal atmospheric pressure, air density $\rho$ is 1.23kg/m$^3$ [15]. The lateral pressure drag coefficient $C_{dp}$ is 1.2, and the skin friction drag coefficient $C_{df}$ is 0.003 (Reynolds number of $5 \times 10^6$) [2].

![Image of graphs showing force against angle of attack with different velocities](image)

Figure 3. The presentation of how the drag force and the lift force at three different flying velocities influenced the attack angle. Drag (Velocity) represented the drag force at three different flying velocities, and Lift (Velocity) represented the lift force at three different flying velocities. (a) male javelin; (b) female javelin.

Comparing the computer-estimated ranges with the measured ones

This study collected the release parameters (Table 1) from other studies in which the javelin ranges of real throw were measured [6,16]. Then these release parameters were quoted in this study to estimate the throw ranges of the male AONR javelin and HNR javelin and that of the female AL javelin. The reason for doing so is to enhance the accuracy of the computer estimation. After simulation and estimation, the result showed that the average relative error between the estimated AONR javelin range and the measured one was 4.2%, while the average relative error between the estimated HNR javelin range and the measured one was 6.8%. For AONR javelin throw, using the release parameters from Best et al. [16] in estimating the throw range created smaller estimation error than using the parameters from Mero et al. [6]. On the other hand, for HNR javelin throw, using the release parameters from Best et al. in estimating the throw range created larger estimation error than using the parameters from Mero et al. [16]. For the AL javelin throw, using the parameters a from Best et al. [16] in estimating the throw range created very tiny error with the average being 2.5%; therefore, the credibility of the estimated range for the AL javelin could be trusted.
Table 1. Analysis of and comparison between male and female in the throwing range

<table>
<thead>
<tr>
<th>Sex</th>
<th>Sex</th>
<th>$V_s$ (ms$^{-1}$)</th>
<th>$z_s$ (m)</th>
<th>$\beta$ (°)</th>
<th>$\phi_0$ (°)</th>
<th>$R^*$ (m)</th>
<th>$\lambda$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M1</td>
<td>M</td>
<td>30.4</td>
<td>1.86</td>
<td>33.5</td>
<td>-2.5</td>
<td>87.42</td>
<td>89.05</td>
</tr>
<tr>
<td></td>
<td>M2</td>
<td>M</td>
<td>29.2</td>
<td>1.88</td>
<td>32.0</td>
<td>-8.0</td>
<td>83.22</td>
<td>81.46</td>
</tr>
<tr>
<td></td>
<td>M3</td>
<td>M</td>
<td>29.2</td>
<td>1.86</td>
<td>32.0</td>
<td>-5.5</td>
<td>81.62</td>
<td>81.37</td>
</tr>
<tr>
<td></td>
<td>K1</td>
<td>M</td>
<td>29.2</td>
<td>1.83</td>
<td>30</td>
<td>-3</td>
<td>88.18</td>
<td>78.88</td>
</tr>
<tr>
<td></td>
<td>K2</td>
<td>M</td>
<td>28.9</td>
<td>1.81</td>
<td>31</td>
<td>-7</td>
<td>86.60</td>
<td>78.70</td>
</tr>
<tr>
<td></td>
<td>K3</td>
<td>M</td>
<td>29.5</td>
<td>1.88</td>
<td>33</td>
<td>10</td>
<td>83.38</td>
<td>82.66</td>
</tr>
<tr>
<td></td>
<td>M1</td>
<td>F</td>
<td>24.2</td>
<td>1.66</td>
<td>34.5</td>
<td>1.5</td>
<td>57.22</td>
<td>57.68</td>
</tr>
<tr>
<td></td>
<td>M2</td>
<td>F</td>
<td>24.6</td>
<td>1.68</td>
<td>39.0</td>
<td>1.0</td>
<td>59.34</td>
<td>61.33</td>
</tr>
<tr>
<td></td>
<td>M3</td>
<td>F</td>
<td>24.6</td>
<td>1.62</td>
<td>38.0</td>
<td>6.5</td>
<td>62.32</td>
<td>60.70</td>
</tr>
<tr>
<td></td>
<td>M4</td>
<td>F</td>
<td>24.2</td>
<td>1.72</td>
<td>33.5</td>
<td>13.5</td>
<td>58.28</td>
<td>56.24</td>
</tr>
</tbody>
</table>

M1, M2, M3, F1, F2, F3, F4 were the release parameters adopted from Best et al. [16]. K1, K2, K3 were the release parameters adopted from Mero et al. [6]. My data represented the ranges estimated by computer ($\lambda$ = 0m/s, $\delta$ = 0 deg/s), and e meant the relative error. The average relative error: AONR(e)=4.2%, HNR(e)=6.8%, AL(e)=2.5%.

Table 2. The estimated optimal release conditions at different release speeds for AONR javelin

<table>
<thead>
<tr>
<th>$V_s$ (m/s)</th>
<th>$R$ (m)</th>
<th>$\beta$ (°)</th>
<th>$\delta_0$ (°)</th>
<th>$\phi_0$ (°)</th>
<th>$\Phi$ (s)</th>
<th>$\delta_1$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>85.85</td>
<td>41.9</td>
<td>32.1</td>
<td>-9.8</td>
<td>4.17</td>
<td>-59.2</td>
</tr>
<tr>
<td>30</td>
<td>91.43</td>
<td>42.2</td>
<td>33.6</td>
<td>-8.6</td>
<td>4.25</td>
<td>-56.5</td>
</tr>
<tr>
<td>31</td>
<td>97.23</td>
<td>42.5</td>
<td>35.3</td>
<td>-7.2</td>
<td>4.47</td>
<td>-54.9</td>
</tr>
<tr>
<td>32</td>
<td>103.22</td>
<td>42.8</td>
<td>36.6</td>
<td>-6.2</td>
<td>4.55</td>
<td>-52.4</td>
</tr>
<tr>
<td>33</td>
<td>109.37</td>
<td>43.0</td>
<td>36.8</td>
<td>-6.2</td>
<td>4.79</td>
<td>-50.7</td>
</tr>
</tbody>
</table>

Table 3. The estimated optimal release conditions at different release speeds for HNR javelin ($\lambda$ = 0m/s, $\delta$ = 0 deg/s)

<table>
<thead>
<tr>
<th>$V_s$ (m/s)</th>
<th>$R$ (m)</th>
<th>$\beta$ (°)</th>
<th>$\delta_0$ (°)</th>
<th>$\phi_0$ (°)</th>
<th>$\Phi$ (s)</th>
<th>$\delta_1$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>92.26</td>
<td>38.3</td>
<td>30.5</td>
<td>-7.8</td>
<td>4.32</td>
<td>-41.1</td>
</tr>
<tr>
<td>30</td>
<td>98.75</td>
<td>37.2</td>
<td>31.7</td>
<td>-5.5</td>
<td>4.48</td>
<td>-39.3</td>
</tr>
<tr>
<td>31</td>
<td>105.80</td>
<td>37.0</td>
<td>33.0</td>
<td>-4.0</td>
<td>4.65</td>
<td>-38.3</td>
</tr>
<tr>
<td>32</td>
<td>112.93</td>
<td>36.8</td>
<td>33.5</td>
<td>-3.3</td>
<td>4.79</td>
<td>-37.1</td>
</tr>
<tr>
<td>33</td>
<td>120.49</td>
<td>37.3</td>
<td>34.0</td>
<td>-3.3</td>
<td>4.99</td>
<td>-36.8</td>
</tr>
</tbody>
</table>

Table 4. The estimated optimal release conditions at different release speeds for AL javelin ($\lambda$ = 0m/s, $\delta$ = 0 deg/s)

<table>
<thead>
<tr>
<th>$V_s$ (m/s)</th>
<th>$R$ (m)</th>
<th>$\beta$ (°)</th>
<th>$\delta_0$ (°)</th>
<th>$\phi_0$ (°)</th>
<th>$\Phi$ (s)</th>
<th>$\delta_1$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>59.41</td>
<td>42.0</td>
<td>32.0</td>
<td>-10.0</td>
<td>3.43</td>
<td>-61.6</td>
</tr>
<tr>
<td>25</td>
<td>64.04</td>
<td>42.4</td>
<td>33.1</td>
<td>-9.3</td>
<td>3.57</td>
<td>-57.7</td>
</tr>
<tr>
<td>26</td>
<td>68.89</td>
<td>42.6</td>
<td>34.4</td>
<td>-8.2</td>
<td>3.70</td>
<td>-53.5</td>
</tr>
<tr>
<td>27</td>
<td>73.96</td>
<td>43.0</td>
<td>36.0</td>
<td>-7.0</td>
<td>3.86</td>
<td>-50.5</td>
</tr>
<tr>
<td>28</td>
<td>79.23</td>
<td>43.0</td>
<td>36.0</td>
<td>-7.0</td>
<td>3.99</td>
<td>-48.4</td>
</tr>
</tbody>
</table>
Optimal release angles for AONR, HNR, and AL javelins

When the release speed $V_r$ ranged from 29 m/s to 33 m/s, the optimal release angles for AONR, HNR, and AL javelin throw was discussed here. The estimation result demonstrated that for AONR javelin throw, the optimal release angle ranged from $41.9^\circ$ to $43^\circ$, which was close to the release angle $43^\circ$ ($d = 25.7cm$, $\delta = 43^\circ$) estimated by Soong [2]. For HNR javelin throw, the estimated optimal release angle ranged from $37.0^\circ$ to $38^\circ$, which was close to the average release angle $36^\circ$ ($\delta = 37^\circ$) measured by Whiting et al. [5] in their experiment ($n=85$).

It was also found that when the release speed was within the range from 29 m/s to 33 m/s, the range for AONR javelin throw was shorter than that using HNR by 7.5% to 10.2%, while the optimal release angle increased by 8.6% to 13.3%. The main reason for this phenomenon was that AONR javelin had longer $d$, the distance from COM to COP of the javelin, than HNR javelin [1].

On the other hand, the estimated optimal release angle for the AL javelin ranged from $42^\circ$ to $43^\circ$, which was larger than $36.3^\circ$, the average of the four measured release angles (F1-F4 in Table 1) by 5.7$^\circ$ to 6.7$^\circ$ [16]. The possible reason was that the AL javelin had longer $d$ than the javelin used by F1, F2, F3, and F4 (Table 1).

Table 5. The estimated optimal release conditions for the male and the female javelin world records ($\lambda = 0m/s$, $\delta = 0 deg/s$)

<table>
<thead>
<tr>
<th>Sex</th>
<th>$d$</th>
<th>World Record</th>
<th>$Z_o$</th>
<th>$V_r$ (m/s)</th>
<th>$\beta$ (°)</th>
<th>$\phi_o$ (°)</th>
<th>$\delta$</th>
<th>$\Phi$ (s)</th>
<th>$\delta_1$ (°)</th>
<th>$H_{max}$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>AONR</td>
<td>98.48</td>
<td>1.88</td>
<td>31.213</td>
<td>42.8</td>
<td>-7.3</td>
<td>35.5</td>
<td>4.46</td>
<td>-54.0</td>
<td>25.18</td>
</tr>
<tr>
<td>M</td>
<td>HOR</td>
<td>98.48</td>
<td>1.88</td>
<td>29.935</td>
<td>37.3</td>
<td>-5.5</td>
<td>31.8</td>
<td>4.45</td>
<td>-38.5</td>
<td>19.41</td>
</tr>
<tr>
<td>F</td>
<td>AL</td>
<td>72.28</td>
<td>1.66</td>
<td>26.672</td>
<td>42.6</td>
<td>-8.00</td>
<td>34.6</td>
<td>3.79</td>
<td>-51.1</td>
<td>18.53</td>
</tr>
</tbody>
</table>

$Z_o$ was quoted from Table 1.

Figure 4. The flying trajectories for the javelin world records, portrayed by the computer. The data on release parameters were quoted from Table 5.

Figure 5. The moments relative to the COM of the AONR, HNR and AL javelins. The data on release parameters were quoted from Table 5.

Optimal release conditions for world record holders at zero wind speed

The researcher attempted to find for the male world record holder the optimal attack angle and release angle at different release speeds when the grip height at release is 1.88m. Based on the previous test with computer programs, the researcher
established the possible range for the release angle \( \beta \) and the attack angle \( \phi \) at different release speeds; \( \beta \) ranged from 25\(^\circ\) to 45\(^\circ\) and \( \phi \) ranged from 10\(^\circ\) to -10\(^\circ\). The angle interval for \( \beta \) and \( \phi \) was assumed to be 0.25\(^\circ\), and then the throw range for each angle interval was calculated. From the collection of calculated ranges emerged the maximum range and its release angle and attack angle.

It was found that when the male world record holder threw AONR javelin with the wind speed and angular velocity being zero and the grip height at release being 1.88m (Table 1), he could reach the world record of 94.48m under the optimal release conditions---the release angle being 42.8\(^\circ\), the attitude angle being 35.5\(^\circ\), the attack angle being -7.3\(^\circ\), and the release speed being 31.213m/s. On the other hand, when the male world record holder threw the HNR javelin with the wind speed and angular velocity being zero and the grip height at release being 1.88m (Table 1), he could reach the world record of 94.48m under the optimal release conditions---the release angle being 37.3\(^\circ\), the attitude angle being 31.8\(^\circ\), the attack angle being -5.5\(^\circ\), and the release speed being 29.935m/s. As for the female holder, it was found that when the female world record holder threw the AL javelin with the wind speed and angular velocity being zero and the grip height at release being 1.66m (Table 1), she could reach the world record of 72.28m under the optimal release conditions---the release angle being 42.6\(^\circ\), the attitude angle being 34.6\(^\circ\), the attack angle being -8.00\(^\circ\), and the release speed being 26.672m/s. Figure 4 presented the flying trajectories for AONR, HNR, and AL javelins.

In this paragraph, the issue of the pitching moment for the javelins of three different kinds was discussed. Table 5 presented the data on optimal javelin release parameters for the male and female world holders. With the data, the study calculated the pitching moment of COM (Fig. 5) for AONR javelin, HNR javelin and AL javelin (\( \lambda = 0 \text{m/s} \cdot \delta = 0 \text{deg/s} \)) in the flying process. It was found that AONR javelin and AL javelin had the same pattern for their pitching moment,
which began with single wave negative and turned into single wave positive. The pitching moment with the pattern of the single wave negative was able to make the tip of the flying javelin rotate toward the ground. The curve for the pitching moment of HNR javelin was composed of three negative waves, with the peak wave higher than the other two. However, the peak value was lower than those for AONR and AL javelins. The main reason was that HNR javelin had shorter distance from the COM to COP than AONR and AL javelins.

Besides, the issue of the relationship between attack angle and the distance from the COM to COP was dealt with. It was found that the shorter distance from the COM to COP caused HNR javelin's curve for attack angle and attitude angle to be different from the curves of AONR and AL javelins. Fig.6a-c demonstrated that the attack angle is negative somewhere and positive elsewhere. The variation to COP caused HNR javelin’s curve for attack angle and attitude angle to be different from the curves of AONR and AL javelins. The main reason was that HNR javelin had shorter distance from the COM to COP than AONR and AL javelins. The main reason was that HNR javelin had shorter distance from the COM to COP than AONR and AL javelins.

The influence of wind speed on the javelin range

![Graph showing the influence of wind speed on the javelin range.](image)

The researcher conducted the estimation of the javelin range achievable for the male world record holder under the optimal release conditions (Table 1) and at the wind speed $V_w$ being from -6 m/s to 6 m/s (Fig.7). It was found that tail wind of 3m/s was most advantageous to AONR javelin throw, which increased the range by 0.13m. On the other hand, HNR javelin was more subject to the wind speed, and head wind of -4m/s was most advantageous to HNR javelin throw, which increased the range by 3.16 m. The same result can be found in throwing discus in head wind, which increases the discuss range (Chiu, 2009). As for the AL javelin throw, the tail wind of 2m/s can increase the range by 0.06m. From the above findings, it can be concluded that the AONR javelin and AL javelin have longer $d$, which is a disadvantage when the javelin is thrown in the head wind. Next, HNR javelin has shorter $d$, which can increase the range with the aid of the lift force from the tail wind.

**Reference**


**Appendix A**

$D_{f_k}$ is the equation for the friction drags in the time point $t_k$, and $D_{p_k}$ is the equation for pressure drags [2]:

$$D_{f_k} = \frac{1}{2} \rho C_d S \frac{V_k^2}{2} \cos^2(\phi \cdot \Omega_k)$$  \hspace{1cm} (19)

$$D_{p_k} = \frac{1}{2} \rho C_p S \frac{V_k^2}{2} \sin^2(\phi \cdot \Omega_k)$$  \hspace{1cm} (20)

Incorporating Eq.19-20, $D_{f_k}$ and $D_{p_k}$ into Eq.21 and Eq.22 [15] can result in Eq.4 and Eq.5. Soong [2] listed Eq.5, Eq.6, Eq.19 and Eq.20 in his study, but didn’t elaborate Eq.21 and Eq.22. This study presented Eq.21 and Eq.22 to explain the origin of Eq.4 and Eq.5.
\[ D_k = D_{p_k} \sin(\mathbf{x} \cdot \Omega_k) + D_f \cos(\mathbf{x} \cdot \Omega_k) \]  
\[ L_k = D_{p_k} \cos(\mathbf{x} \cdot \Omega_k) + D_f \sin(\mathbf{x} \cdot \Omega_k) \]