1. Introduction

Traditional vibration energy harvesters, such as the cantilever type [1] or spring-supported proof mass type [2], are designed to harvest vibration energy at single resonant frequency. There is a growing need of harvesting broadband, low frequency, ambient vibration energy for wireless sensor node applications. Bistable mechanisms (BMs) have been shown to have a high potential for broadband vibration energy harvesting [3]. Bistable energy harvesters can be devised to harvest broadband, low frequency ambient vibration energy due to their nonlinear spring softening behaviors [4].

Nonlinearity has been exploited to increase the power output of the monostable, Duffing oscillators or impact type vibration energy harvesters. Due to the interwell oscillations if the BMs, the high velocity of the proof mass of the bistable energy harvester may generate more electrical power than the monostable energy harvesters. The interwell oscillations can occur given the amplitude of the vibration source is large enough to excite the snap-through behaviors of bistable energy harvesters, so that they can efficiently capture more low-frequency vibration energy than the linear, resonant oscillators.

In this paper, a BM for broadband vibration energy harvesting is analyzed. Approximate analytical solutions are obtained by the harmonic balance method. Frequency response of the system is examined. Given the interwell oscillations, the BM is shown to have a much larger bandwidth than the monostable, Duffing oscillator. Experiments are carried out to verify the key features of the bistable energy harvesters.

2. Design

2.1 Principle

Fig. 1(a) is a schematic of a bistable energy harvester. A Cartesian coordinate system is also shown in the figure. Two springs are joined by a slider, which can move along a guide rod. The BM is symmetric with respect to the x axis. The spring has an inclination angle $\theta$. A permanent magnet can be glued to the slider and placed under a conducting coil to facilitate electromagnetic induction for energy harvesting. The assembly of the magnet and the slider is termed as the shuttle mass of the BM.

Schematics of the BM at two stable equilibrium positions, $P_a$ and $P_c$, are shown in Fig. 1(a) and (b), respectively. The BM is initially...
at the first equilibrium position $P_a$ (see Fig. 1(a)). Upon the application of a force $F$ to the shuttle mass in the $x$ direction, the springs are compressed. When the compressive force in the spring increases to a maximum at a certain displacement of the shuttle mass, the mechanism snaps towards its second stable position $P_c$ (see Fig. 1(b)). When the direction of the force $F$ is reversed, the shuttle mass is moved along the $-x$ direction. As the compression of the spring reaches its maximum, the mechanism snaps towards its first stable position $P_a$. The unstable equilibrium positions, $P_b$ of the BM is indicated in Fig. 1(c).

2.2 Lumped model

Fig. 2 A model of the bistable energy harvester

Fig. 3 f-d curves of forward and backward motions of the BM

where the overdot denotes the derivative with respect to time $t$. In this investigation, the value of the spring constant of the springs is taken as 100 mN/mm. The original length of the springs is 60.83 mm. The inclination angle $\theta$ is 9.46°. Fig. 3 shows f-d curves of the forward motion, shown in the sequence of Fig. 2(a-b), and backward motion, shown in the sequence of Fig. 2(b-a), of the BM based on finite element analyses. As seen in Fig. 3, positions $x = 0$ and $x = 10$ are the first and the second stable equilibrium positions of the BM, respectively. The reaction forces for the forward and backward motions can be fitted by a 4th order function, $f(x)$, as

$$f(x) = k_1 x^4 + k_2 x^3 + k_3 x^2 + k_4 x$$

where $k_1 = 0.000007$, $k_2 = 0.027171$, $k_3 = -0.819697$, and $k_4 = 5.473274$. These equations can be non-dimensionalized by defining

$$\omega = \frac{\Omega}{L_0 m}, \quad \tau = \omega t$$

Substituting the dimensionless frequency $\omega$, dimensionless time $\tau$, dimensionless displacement $x$, and dimensionless current $i$ into Eq. (1) and using Eq. (2), we obtain the dimensionless equations as

$$x'' + \mu x' + \alpha x + \beta x^3 + \gamma x^5 = 0$$

$$i'' + \mu_e i' - \zeta = 0$$

where $\mu = d / \sqrt{mk_1}$ is the dimensionless mechanical damping, $\zeta = \psi / \sqrt{L_0 k_1}$ is the dimensionless transducer constant, $A = F k_1^2 L_0^2$ is the dimensionless, and $\mu_e = (R_L + R_e) \sqrt{m/k_1 / L_0}$ is the dimensionless electrical damping. The prime denotes a derivative with respect to dimensionless time $\tau$.

3. Analysis

In this investigation, a Duffing oscillator (DO) is selected as a representative of a monostable energy harvester to be compared with the BM. The purpose is to demonstrate that the bandwidth of the BM is much higher than that of a DO with the same footprint. The frequency response of the bistable energy harvester and the Duffing-type energy harvester to a harmonic loading is analyzed by the method of harmonic balance. The analysis follows a procedure reported by Stanton et al. [5]. The dimensionless equations of motion for the DO-type energy harvester can be expressed as

$$x'' + \mu x' + \alpha x + \beta x^3 + \gamma x^5 = 0$$

$$i'' + \mu_e i' - \zeta = 0$$

For fair comparison, the BM and the DO have similar normalized f-d curves. When the spring of the mechanism shown in Fig. 1(a) has a f-d relation of Fig. 4, an original length of 863.31 mm and an inclination angle $\theta$ of 9.46°, the mechanism behaves similar to a DO. The mechanism with the above f-d relation, original length and inclination angle is considered as a DO, hereafter. Fig. 5 shows the normalized f-d curves of the BM and the DO. They have approximately the same value of maximum force in the same displacement range. When the values of and of Eq. 5(a) are taken as 0.037983 and 0.027203, respectively, the DO has the normalized f-d curve shown in Fig. 5.
The oscillation amplitude of the nonlinear systems of Eqs. (4) and (5) may vary in a wide range. Therefore, the classical harmonic balance method can be used to solve the nonlinear oscillation systems. A solution to Eq. (4) can be assumed as

\[ x = c_1(\tau) + a_1(\tau) \sin \omega \tau + b_1(\tau) \cos \omega \tau \]  

\[ \dot{x} = a_2(\tau) \sin \omega \tau + b_2(\tau) \cos \omega \tau \]  

(6a)

(6b)

Assuming slowly varying coefficients, i.e. \( \dot{c}_1 = \ddot{a}_1 = \ddot{b}_1 = 0 \), substituting the necessary derivatives of \( x, \dot{x} \) and \( \ddot{x} \) into Eq. (4), neglecting higher harmonics and balancing the constant terms and the coefficients of \( \sin \omega \tau \) and \( \cos \omega \tau \), the steady state equations for the bistable energy harvester are obtained. Solutions of the Duffing-type energy harvester can be obtained similarly. The frequency response can be determined by finding the real roots of the steady state equations. The steady state amplitude of current, \( I \), is given as

\[ I = \sqrt{a_1^2 + b_1^2} \]  

(7)

The average power through the electrical impedance can be written as

\[ P_{avg} = \frac{I^2}{2\mu_e} \]  

(8)

4. Results

Depending on the level of the excitation amplitude \( A \), the BM can exhibit intrawell, low-energy oscillations, or interwell, high-energy oscillations. When undergoing interwell, high-energy oscillations, the BM may exhibit a larger bandwidth than the DO. In order to excite the interwell response of the BM at the frequencies near its fundamental vibration modes, the value of \( A \) is taken as 0.39.

To determine the fundamental system response, the roots of the steady state equations were computed for the parameters \( \mu = 0.08 \), \( \mu_e = 0.01 \), \( \zeta = 0.9 \) and \( A = 0.39 \) for both the BM and the DO. At each excitation frequency \( \omega \), the real roots of the steady state equations were solved numerically. Fig. 6 is the response amplitudes of the BM and the DO. Fig. 6(a) shows the response amplitude of the BM. The response curves of \( c_1 = P_3 \) and \( c_1 \neq P_3 \) correspond to the oscillations around the unstable equilibrium position of the BM and otherwise, respectively. As shown in Fig. 6(a), the BM can sustain high-energy oscillations in the frequency range of approximately 0.8 to 2.0 where the response amplitude is higher than 1. The high-energy oscillations of the DO persist in the frequency range of approximately 0.7 to 1.1 where the response amplitude is higher than 1. The BM exhibits a threefold increase in the frequency range of high-energy oscillations compared to the DO. Under the considered excitation level, the BM is capable of sustaining high-energy oscillations in a wide frequency range. Broadband vibration energy harvesting can be achieved by the interwell, high-energy oscillations of the BM.

5. Summary

The bandwidth of the BM for vibration energy harvesting is shown to be much larger that the DO by harmonic balance analysis. With similar f-d characteristics, the BM has a threefold increase in the frequency range of high-energy oscillations compared to the DO.

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REFERENCES


