Design of a crab-like bistable mechanism for nearly equal switching forces in forward and backward directions

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Abstract

A compliant bistable mechanism with a crab-like structure for nearly equal switching forces in the forward and backward directions is developed. It consists of four crab-leg-like beams and a shuttle mass. The crab-leg like beam gives more design freedom than straight beam or curved beam seen in the traditional bistable mechanisms. This design flexibility is exploited here for the design of a bistable mechanism with nearly equal switching forces in the forward and backward directions. Key parameters affect the force-displacement characteristics of the bistable mechanism are identified. The performance of the mechanism is confirmed by experiments. Such a mechanism can be applied in nonvolatile memory devices with two logical levels “1” and “0” assigned to the two stable states which are separated by nearly equal switching forces in back and forth directions.

Keywords: Bistable; crab-like structure; switching force.

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1. Introduction

Given the advantages of ease of open-loop control, zero power consumption to stay in stable equilibrium states and insensitivity to noise [1], compliant bistable mechanisms (CBMs) have potential applications in nonvolatile memory elements [2], micro switch or micro relays [3], projection displays [4], threshold accelerometers [5], etc. In the aspect of design of CBMs, force-displacement (f-d) and energy-displacement relations are often considered as the design metrics. A CBM with nearly equal switching forces in back and forth directions (symmetric force output) can be applied in nonvolatile memory devices with two logical levels “1” and “0” assigned to the two stable states. The symmetric force output of the CBM can be applied in threshold accelerometers to achieve two sensing directions along the sensing axis. For bistable micro switches driven by comb-drive type actuators, force symmetry may be advantageous since the force developed by the actuator is proportional to the applied electric potential which may be
limited by the conventional complementary metal-oxide-semiconductor (CMOS) technology.

Force asymmetry with respect to the sign of deflection of CBMs is usually present in the literatures [6-12]. The traditional straight beam type design [6, 9, 13] and curved beam type design [10-12] restrict the degree of freedom in design of CBMs and often render them with unequal switching forces in the forward and backward directions during their operation. Liu et al. [14] reported a CBM with symmetric force output based on pre-shaped beams. A pre-load operation is necessary to bring their structure into initial configuration. Another approach to obtain symmetric force behavior of CBMs is by pre-compression of slender beams [15]. The additional control scheme for the amount of compression introduces design complexity.

The straight beam or curved beam of the CBMs can be sectioned into several segments in order to have the design flexibility to achieve required bistable characteristics. The number and the configuration of the segments have great influence on the switching forces and switching modes [16]. Key configurations of the sectioned segments should be investigated to decrease design complexity and ease the design process. Methods based on optimization approaches [17], analytical equations [18] or inverse static analyses [19] can then be utilized to design the CBMs with specific f-d characteristics. Valentini and Pennestrì [20] developed an analytical model to compute the stiffness of flexure hinges for synthesis of flexure joints. Verotti [21] derived analytical expressions to determine the position of the center of rotation of flexures to assist the design of compliant mechanisms. A bistable mechanism with equal switching forces will also benefit design of tristable mechanisms and multistable mechanisms [22]. Chen et al. [22] utilized a bistable mechanism for providing latching forces, in which large enough latching forces are required for both forward and backward motion.

In this investigation, a crab-like CBM for symmetric force output with respect to the sign of deflection is developed. Fig. 1(a) and (b) shows a crab and a crab-like CBM, respectively. The beams of the CBM are similar to the walking legs of the crab in shape. The five-segmented beams of the CBM gives more design freedom. The force symmetry of the CBM is sought by adjusting the lengths of the five sections of the beam and the inclination angle using an optimization approach. Finite element analyses are utilized to evaluate the f-d characteristics of the CBM. Key design parameters are recognized for efficient design of the CBM with symmetric force output. Experiments are carried out to verify the force symmetry of the CBM.
Fig. 1 Schematic of (a) a typical crab and (b) a crab-like CBM.

Fig. 2 (a) A full model of the CBM. (b) A quarter model of the CBM.
2. Design

2.1 Operational principle

Because the body and leg movements of crabs are almost entirely in the same plane and the high agility of their legs, crab-like mechanisms are well-suited for design of planar motion mechanism [23]. Fig. 1(a) is a sketch of a crab. One of its legs can be dissected into five sections as numbered in the figure. This five sectioned geometry can be mimicked to develop a mechanism for motion generation and control. A schematic of a crab-like CBM with four five-section beams and a shuttle mass is shown in Fig. 1(b). The analogy between the crab leg and the beam of the CBM is indicated by the numbers shown in Fig. 1(a) and (b), respectively. It is assumed the CBM remains parallel to the underlying substrate. Fig. 2(a) is a schematic of the CBM where a shuttle mass is supported by four beams. One end of the beams is fixed to the substrate. A Cartesian coordinate system is also shown in the figure. Each beam consists of one stepped slanted section and two horizontal sections. As shown in Fig. 2(a), the mechanism is designed to move in the $x$ direction when a force $F$ is applied quasistatically at the center of the shuttle mass. Fig. 2(b) is a schematic of a quarter model of the mechanism. The left end of the beam of the quarter model can be represented as a fixed boundary condition. The symmetric plane of the quarter model can be represented as a roller boundary condition. The slanted section has an inclination angle of $\theta$. The device has a thickness of $t$. The four design parameters, $L_1$, $L_2$, $L_3$ and $\theta$, are also indicated in Fig. 2(b). In this investigation, values of $t$, $W_1$, $W_2$ and $W_3$ are specified according to available fabrication methods. Dissimilar to the existing three section beam design of Li and Chen [16], the center section of the CBM is a double stepped beam.

Fig. 3 A typical f-d curve of a CBM.

A typical f-d curve of a CBM is shown in Fig. 3. At the beginning, the shuttle mass rests at its first stable equilibrium position $S_1$. When the shuttle mass is moved quasistatically in the $+x$ direction, the sum of the reaction force at the fixed ends increases initially, attains its maximum value $F_{\text{max}}$, then decreases and the shuttle mass
reaches its unstable equilibrium position \( P \). As the shuttle mass is moved further in the \(+x\) direction, the reaction force turns negative, attains its minimum value \( F_{\text{min}} \), then increases and the shuttle mass reaches its second stable equilibrium position \( S_2 \). While the CBM is at the unstable equilibrium position \( P \), a light perturbation to the CBM could drive the shuttle mass into the stable equilibrium positions \( S_1 \) or \( S_2 \). In the displacement controlled mode of motion, the positive and negative reaction forces mean that the shuttle mass is pushed and pulled in the forward motion of the mechanism, respectively. In the load controlled mode of motion, the load is increased quasistaticaly in the forward motion. When the position \( Q_1 \) is reached, the mechanism will snap-through toward its second equilibrium position with no appreciable change in the load. When the shuttle is moved backward, the mechanism follows the f-d curve reversely in the displacement controlled mode of motion. The insets of Fig. 3 show two stable equilibrium states of the CBM. \( F_{\text{max}} \) and \( F_{\text{min}} \) are the switching forces for the forward and backward motion, respectively. Given \( F_{\text{max}} = -F_{\text{min}} \), force symmetry with respect to the sign of deflection is established.

2.2 Design

Due to the complex geometry of the CBM, an analytical f-d solution may not be obtained to aid in the design of the CBM. Therefore, the optimum design approach is adopted in this investigation. \( L_1, L_2, \) and \( L_3 \), and the inclination angle \( \theta \) of the CBM are selected as the design variables. The objective functions of the optimization problem are

\[
\begin{align*}
\text{Min} & \quad \left| \frac{F_{\text{max}}}{f_1} - 1 \right| \\
\text{Min} & \quad \left| \frac{F_{\text{min}}}{-f_2} - 1 \right| \\
\text{Min} & \quad \left| \frac{S_2}{d} - 1 \right|
\end{align*}
\]

where \( f_1 \) and \( f_2 \) are the target values of the output forces of the CBM, and \( d \) is the target value of the second equilibrium position. The purpose of Eq. (1) and Eq. (2) is to attain desired output force. For symmetric output force, \( f_1 \) should be equal to \( f_2 \), that is \( f_1 = f_2 = f \) and \( F_{\text{max}} = -F_{\text{min}} = f \). The sign of \( F_{\text{max}} \) and \( F_{\text{min}} \) must be opposite and their absolute values must be as close to \( f \) as possible. The formulation of Eq. (3) specifies the distance between the first and the second stable equilibrium position. The objective functions of Eq. (1) and Eq. (2) are formulated in a way to have the robustness for optimal solutions of \( F_{\text{max}} \) and \( F_{\text{min}} \). The optimization design method presented here can be utilized to design a CBM with specified \( F_{\text{max}}, F_{\text{min}} \) and \( S_2 \) for versatile applications of CBMs.

Table 1. Lower and upper bounds on the design variables.
<table>
<thead>
<tr>
<th>Variables</th>
<th>Lower bound (mm)</th>
<th>Upper bound (mm)</th>
</tr>
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<tbody>
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<td>12</td>
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<tr>
<td>$L_2$</td>
<td>5</td>
<td>10</td>
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<tr>
<td>$L_3$</td>
<td>25</td>
<td>40</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$3^\circ$</td>
<td>$8^\circ$</td>
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</table>

The values of $f$ and $d$ are taken as 1500 mN and 6.5 mm, respectively, in this investigation. The values of $W_1$, $W_2$ and $W_3$ are specified as 0.8 mm, 0.8 mm and 10 mm, respectively. The out-of-plane thickness of the mechanism $t$ is taken as 5 mm. The number of the design variables is 4. Table 1 lists the lower and upper bounds on the design variables. The shuttle mass has its dimensions indicated in Fig. 2(a). This two-objective optimization of the CBM is solved by a nondominated sorting genetic algorithm [24]. In this algorithm, multiple objectives can be aggregated into a single objective function using nondominated sorting procedure [25]. Due to the geometry complexity and nonlinear behaviors of the CBM, the output force versus displacement curve of the CBM is analyzed by finite element method during the optimization process. The finite element package ABAQUS [26] is used in the finite element analyses.

![Fig. 4 A mesh of the half model of the CBM.](image)

Due to symmetry, a half model of the mechanism is considered in the finite element analyses. Fig. 4 is a mesh of the half model. In order to represent the symmetry condition at the symmetric plane, its displacement in the $y$ direction is constrained. Fixed boundary conditions are applied to the anchors of the mechanism. A displacement is applied in the $x$ direction and $-x$ direction to the shuttle mass for the forward motion and the backward motion of the mechanism, respectively. The material of the mechanism is assumed to be polyoxymethylene (POM). The material is assumed to be linear elastic and isotropic. The Young’s modulus and the Poisson’s ratio of the material are 2.15 GPa and 0.38, respectively. The element type of the finite element model is 4-node element CPE4R. This second-order reduced integration element is very effective in terms of accuracy and is economical because of the reduced number of integration points.
Fig. 5  Population distribution of the 10th, 20th and 45th generation during evolutionary process.

Table 2. The values of the design variables of the optimum design.

<table>
<thead>
<tr>
<th>Variable</th>
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<td>$L_1$</td>
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<tr>
<td>$L_2$</td>
<td>5.65</td>
</tr>
<tr>
<td>$L_3$</td>
<td>34.33</td>
</tr>
<tr>
<td>$\theta$</td>
<td>3.92°</td>
</tr>
</tbody>
</table>

2.3 Optimization

The number of generations and the population of each generation are taken as 45 and 20, respectively, in the optimization process. Fig.5 displays the population distribution of the 10th, the 20th and 45th generation during the evolutionary process. The three coordinates of the figure represent the values of the three objective functions of Eqs. (1-3). As the evolution goes, the individuals in the population converge to the dominant ones. The optimal solution of the 45th generation is marked in the figure. The values of the design variables of the optimum design are listed in Table 2. Fig. 6(a)
shows the f-d curve of the optimum design of the CBM, where $F_{\text{max}} = 1540 \text{ mN}$, $F_{\text{min}} = -1498 \text{ mN}$, and $S_2 = 6.50 \text{ mm}$. Therefore, $F_{\text{min}}$ and $S_2$ have nearly the same values as the target values. $F_{\text{max}}$ is 2.7% higher than the target value. The difference between $F_{\text{max}}$ and $|F_{\text{min}}|$ is nearly 2.7%. Force symmetry with respective to the sign of displacement is achieved within a reasonable margin. Furthermore, the second equilibrium position can be readily attained by this crab-like beam design. In order to avoid yielding of the CBM under loading, the stress in the mechanism should not exceed the typical yield strength, 62 MPa, of the POM material. Fig. 6(b) shows a maximum stress-displacement curve of the design. The peak value of the maximum stress in the CBM is attained just before the mechanism reaches its unstable equilibrium position (see Fig. 6). The maximum stress is 14.64 MPa, well below the typical value of the yield strength of POM material. Fig. 6(c) shows the location of the peak value of the maximum stress in the CBM. The peak value is found in the flexural beam 5 near the shuttle mass.
Fig. 6  Force versus displacement curve (a) and maximum stress curve (b) of the CBM. (c) Location of the peak value of the maximum stress in the CBM.

3. Fabrication and testing

Prototypes of the mechanism were fabricated in order to demonstrate the force symmetry of the crab-line mechanism. The prototypes were engraved by a milling machine from the POM material. The thickness of the POM material is 5 mm. Dimensions of the prototypes are based on the optimum design listed in Table 2. Fig. 7(a) is a photo of a fabricated CBM. Fig. 7(b) shows the measured dimensions of the fabricated device. The lengths and widths are measured by a caliper. The angle $\theta$ is calculated by the inverse of the tangent function where the side lengths are measured by the caliper. Because of wear, the machine tool may have a lower precision. The mechanical resolution and positioning accuracy of the available milling machine are limited. The difference between the design dimensions and measured dimensions can be
attributed the positioning error of the milling machine. Fixture error due to positioning inaccuracies of workpiece and fixture is one of the machining error sources.

Fig. 7 (a) A photo of a fabricated prototype. (b) A schematic of the measured dimensions.
Fig. 8 Snapshots of the forward (a-b) and backward (c-d) motion of the CBM.

Fig. 8(a-b) are snapshots of the CBM moving from the first stable equilibrium position to the second stable equilibrium position by pushing the top surface of the shuttle mass forward. At the event of snap-through of the CBM, the top surface loses contact with the probe (see Fig. 8(b)). By pulling the bottom surface of the shuttle mass backward, the CBM moves from the second stable equilibrium position to the first stable equilibrium position (see Fig. 8 (c-d)). Fig. 9 is a photo of the experimental setup for measurement of the f-d curve of the CBM. The mechanism is mounted on an acrylic plate. A force gauge (DS2-5N, Zhiqu Precision Instruments Co., Ltd., China) is used to apply a force to the CBM and is held by a translation stage. The full scale rating and resolution of the force gauge are 5 N and 0.001 N, respectively. The displacement of the shuttle mass of the CBM is read from a vernier micrometer. The acrylic plate and the translation stage are mounted on an optical table.

Fig. 9 An experimental setup.
Fig. 10  Force versus displacement curves for (a) forward; (b) backward motion.

4. Results and discussions.

Fig. 10(a) and (b) show f-d curves of the CBM bases on experiments and finite element analyses during the forward and backward motion, respectively. The models with the optimum design dimensions and fabricated device dimensions are considered in the finite element analyses. The experiments are repeated 6 times for both forward and backward motion. The error bars shown in the figure indicate the range of the experimental values. Repeatability of stable positions is verified by the experiments. During the experiments, the translation stage with the force gauge on top is moved quasistatically to obtain the f-d curves in the forward motion and backward motion. Initially, the mechanism rests in its first stable equilibrium position. The probe tip of the force gauge is pushed slowly against the top surface of the shuttle mass of the mechanism.
As the displacement increases and the mechanism reaches its unstable equilibrium position, the shuttle mass loses contact with the probe tip and snaps to the second stable equilibrium position (see the discontinuous jump of the experimental results in Fig. 10(a)).

As shown in Fig. 10(a), before the applied force reaches the maximum value, the experimental results are slightly higher than those based on the finite element model with the design dimensions. A fair comparison between the experiment and the finite element model with the measured dimensions is observed. As the probe tip pushes the shuttle mass further, a reduction in the output force indicates the incoming of the snap-through of the CBM (see Fig. 10(a)). In the event of snap-through, the probe tip loses contact with the shuttle mass and the mechanism jumps to its second stable equilibrium position during experiments.

In order to obtain a f-d curve for backward motion, a ring is affixed to the probe tip. Therefore, the shuttle mass can be pulled by the probe. As shown in Fig. 10(b), the force magnitude of the experimental results deviates slightly from that based on the finite element model with the design dimensions. The force magnitude based on the finite element model with the measured dimensions agrees with the experiments. As the probe tip pushes the shuttle mass further, a reduction in the magnitude of the output force indicates the incoming of the snap-through of the CBM. In the event of snap-through, the probe tip loses contact with the shuttle mass and the mechanism jumps to its first stable equilibrium position (see the discontinuous curve of the experimental results in Fig. 10(b)). The experimental \( F_{\text{max}} \) and \( |F_{\text{min}}| \) values are 1398 mN and 1378 mN, respectively. The experimental \( S_2 \) value is 6.7 mm. The difference between the experimental \( F_{\text{max}} \) and \( |F_{\text{min}}| \) is around 1.4 %. 
Perfect symmetry force output in the forward and backward motion of a bistable mechanism can be achieved by compressing a beam with an axial load [27]. The load has to be above the first critical load for buckling. Traditional curved-beam [28] and chevron-type [29] bistable mechanisms possess asymmetric force output with respect to the forward and backward motion. Huang et al. [30] presented a cosine curved beam with multiple reinforced segments for design of a bistable mechanism. Force outputs can be adjusted by modifying the length, width, thickness, element number and position of the beams’ segments. For the pre-fabricated CBM presented in this investigation, the horizontal beam section, indicated by the number 1 in Fig. 1(b), plays a significant role to obtain the equal maximum force magnitude in its forward and backward motion. A slight deviation of the horizontal beam from its leveled position results in a large variation in the f-d curves of the CBM. Fig. 11(a) is a schematic of a quarter model of the CBM, where the inclination angle of beam 1, $\theta'$, is indicated. Fig. 11(b) shows the effects of the inclination angle of beam 1, $\theta'$, on the f-d curves of the CBM. All other parameters of the optimum design of the CBM are kept unchanged. As shown as in Fig. 11(b), as $\theta'$
is decreased from $3^\circ$ to $-3^\circ$, $F_{\text{max}}$ decreases monotonically, but $|F_{\text{min}}|$ increase initially, saturates near $\theta' = 0$, then decreases. It can be seen that a leveled beam 1, $\theta' = 0$, plays an important role to reach $F_{\text{max}} = |F_{\text{min}}|$ for the design of the CBM.

![Graphs showing the effects of geometry parameters on the f-d curve.](https://via.placeholder.com/150)

**Fig. 12** Effects of geometry parameters on the f-d curve.

The effects of other geometric parameters on the f-d curves are investigated to assist in the design of the CBM using the finite element model. In the following investigation, the value of $W_1$ is taken as the same as that of $W_2$. Fig. 12(a) illustrates how the length $L_1$ affects the values of the $F_{\text{max}}$ and $F_{\text{min}}$ of the CBM. Five values of $L_1 / L_3$ are considered by varying $L_1$ with all other parameters kept the same as the optimum design. The solid curve represents the optimum design that is marked with a superscript in the legend of the Fig. 12(a-d). As shown in Fig. 12(a), when $L_1$ is increased, the magnitudes of $F_{\text{max}}$ and $F_{\text{min}}$ are decreased monotonically. The results indicate that as $L_1$ is varied, the characteristics of nearly symmetric force output are not lost. Therefore, $L_1$ has a great influence on the magnitude of the symmetric force.
As shown in Fig. 12(b), when the value of $W_3$ increases while all other parameters are kept the same as the optimum design, the magnitudes of $F_{\text{max}}$ and $F_{\text{min}}$ are increased and then saturated when $W_1/W_3$ reaches a value of 0.08. This result indicates that any $W_1/W_3$ value less than 0.08 has little effect on the magnitudes of $F_{\text{max}}$ and $F_{\text{min}}$. Fig. 12(c) shows that as the value of $L_2/L_3$ increases while the other parameters are kept the same as the optimum design, the value of $F_{\text{max}}$ decreases and the value of $F_{\text{min}}$ has no apparent change. This result indicates that the value of $L_2/L_3$ is not a critical parameter to adjust as the design is relatively close to the symmetric force output region.

Fig. 12(d) shows the effects of $\theta$ on the f-d relation of the CBM. The CBM loses its force output symmetry as the value of $\theta$ deviates from its optimal value while all the other parameters are kept the same as the optimum design. As seen in Fig. 12(d), when the value of $\theta$ is smaller than the optimal value, the magnitudes of $F_{\text{max}}$ and $F_{\text{min}}$ decrease, and the reduction in $|F_{\text{min}}|$ is significantly larger than that of $F_{\text{max}}$. With a larger value of $\theta$ compared to its optimal value, the magnitudes of $F_{\text{max}}$ and $F_{\text{min}}$ increase. Based on the computational results shown in Fig. 12, $L_1$ and $\theta$ have much higher influence on the f-d relation of the CBM compared to the other geometry parameters. In the initial design stage, $L_1$ and $\theta$ can be adjusted to obtain a f-d relation near the design specification. Then, $L_2$, $L_3$ and $W_3$ can be fine-tuned to reach a better design with the $W_1/W_3$ value kept less than 0.08 and $W_1/W_2$ value kept equal to 1.

For the results shown in Fig. 12, the value of $W_1$ is taken as the same as that of $W_2$. The ratio $W_1/W_2$ determines the geometry and the bending stiffness of the flexure of the CBM. The effects of $W_1/W_2$ on the f-d curves of the CBM should be examined. Five values of $W_1/W_2$ are considered by varying $W_1$ with all other parameters kept the same as the optimum design. The solid curve represents the optimum design that is marked with a superscript in the legend of the Fig. 13. When the value of $W_2$ is increased, the magnitudes of $F_{\text{max}}$ are decreased and the magnitudes of $F_{\text{min}}$ appear nearly the same initially, then decreased. The lower half of the f-d curve is distorted toward the unstable equilibrium position while $W_2$ is increased. When $W_2$ is decreased, the magnitudes of $F_{\text{max}}$ are increased and the magnitudes of $F_{\text{min}}$ are decreased. This result indicates that any $W_1/W_2$ value greater than 1, the flexure is stiffer in the forward direction of the CBM, while any $W_1/W_2$ value less than 1, the flexure is softer in the forward direction of the CBM. It appears that any $W_1/W_2$ value different from one, the flexure tends to be softer in the backward direction of the CBM.
Fig. 13  Effects of \( W_1/W_2 \) on the f-d curve.
Fig. 14  Relations between various geometry parameters of the CBM.

An atlas relating the key geometry parameters plays a pivotal role in design of the CBM with symmetric force output. Fig. 14 shows the relations between various geometric parameters of the CBM. The values of the symmetric force output are also indicated in the figure. Fig. 14 shows the computational results where the value of \( L_2 / L_1 \) is plotted as a function of \( L_1 / L \) for the CBM with various sets of \( L / H \), \( W_1 / W_2 \), and \( t / H \) values. The dimensions represented by \( L \) and \( H \) are indicated in Fig. 2(b). For the case of \( L / H = 12 \), \( W_1 / W_2 = 0.08 \), and \( t / H = 1.6 \) (see Fig. 14(a)), the equation of the line of a least square fit is \( 346.0 / 322.5 / 132 = L_2 / L_1 \). For the other three cases shown in Fig. 14(b-d), the equations of the linear fit are \( 444.0 / 843.5 / 132 = L_2 / L_1 \), \( 638.0 / 335.6 / 132 = L_2 / L_1 \), and \( 524.0 / 428.5 / 132 = L_2 / L_1 \), respectively. As can be seen in these figures, larger values of \( L_1 \) and \( L_2 \) contribute to the decrease in the
symmetric force value. The result shown in Fig. 14 is a small fraction of the design space of the CBM. It provides some guidelines of the selection of the geometry parameters in the initial design stage.

A less restrictive initial condition of the optimization design problem of the CBM is also considered, where \( W_1 = 0.8 \text{ mm}, W_2 = 0.9 \text{ mm}, \) and the value of \( \theta \) has a wider range. Table 3 lists the lower and upper bounds on the design variables. The target values of \( f \) and \( d \) are taken as 1500 mN and 6.5 mm, respectively. The value of \( W_3 \) is specified as 10 mm. The out-of-plane thickness of the mechanism \( t \) is taken as 5 mm. The shuttle mass has its dimensions indicated in Fig. 2(a). The number of generations and the population of each generation are taken as 45 and 20, respectively. The values of the design variables of the optimum design are \( L_1 = 7.92 \text{ mm}, L_2 = 6.76 \text{ mm}, L_3 = 30.94 \text{ mm}, \) and \( \theta = 3.5^\circ \). Values of \( F_{\text{max}}, F_{\text{min}} \) and \( S_2 \) of the optimum design of the CBM are, 1689 mN, -1273 mN, and 6.2 mm, respectively. \( F_{\text{max}} \) and \( F_{\text{min}} \) are 13% higher and 15% lower than the target value, respectively. The difference between \( F_{\text{max}} \) and \( |F_{\text{min}}| \) is 416 mN. Force symmetry is not achieved within a reasonable margin by the less restrictive initial condition of the optimization design problem.

### Table 3. Bounds on the design variables for a less restrictive initial condition.

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<tr>
<th>Variables</th>
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<th>Upper bound (mm)</th>
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<tbody>
<tr>
<td>( L_1 )</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>( L_2 )</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>( L_3 )</td>
<td>25</td>
<td>50</td>
</tr>
<tr>
<td>( \theta )</td>
<td>3(^\circ)</td>
<td>10(^\circ)</td>
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</table>

Combination of Beam 1 and Beam 2 in the 5-segmented beam design, see Fig. 1(b), may be taken as a flexural member with a constant cross-section and a kinematic discontinuity. A case with a constant cross-section curved flexural member should be examined and compared to the five-segmented beam design of the CBM. Fig. 15 is a schematic of a curved flexural member of a CBM. The shape of the curved beams is

\[
y_r = \frac{H_c}{2} \left( 1 - \cos \frac{\pi x_r}{L_c} \right)
\]

where \((x_r, y_r)\) is the position vector with the reference origin at the left end of the curved beams. \( L_c \) and \( H_c \) are the span and apex height of the curved beams, respectively. Table 4 lists the lower and upper bounds on the design variables. The target values of \( f \) and \( d \) are taken as 1500 mN and 6.5 mm, respectively. The values of \( W_1 \) and \( W_3 \) are specified as 0.8 mm and 10 mm, respectively. The out-of-plane thickness of the mechanism \( t \) is taken as 5 mm. The shuttle mass has its dimensions indicated in Fig. 2(a). The number of generations and the population of each generation are taken as 45 and 20, respectively. The values of the design variables of the optimum design are \( L_c = 17.5 \text{ mm}, H_c = 1.95 \text{ mm}, L_3 = 46.2 \text{ mm}, \) and \( \theta = 5.03^\circ \). Values of \( F_{\text{max}}, F_{\text{min}} \) and \( S_2 \) of
the optimum design of the CBM are 1478 mN, $F_{\text{min}} = -445$ mN, and $S_2 = 6.5$ mm. $S_2$ has nearly the same value as the target value. $F_{\text{max}}$ and $F_{\text{min}}$ are 1% and 70% lower than the target value, respectively. The difference between $F_{\text{max}}$ and $|F_{\text{min}}|$ is 1033 mN. Given the large difference between $F_{\text{max}}$ and $|F_{\text{min}}|$, force symmetry may not be achieved by the cosine curved beam design.

Table 4. Bounds on the design variables of a curved beam flexure design.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Lower bound (mm)</th>
<th>Upper bound (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_c$</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>$H_c$</td>
<td>0.5</td>
<td>4</td>
</tr>
<tr>
<td>$L_3$</td>
<td>25</td>
<td>50</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$0^\circ$</td>
<td>$6^\circ$</td>
</tr>
</tbody>
</table>

![Fig. 15 A schematic of a curved flexural member of a CBM.](image)

In this investigation, the optimization approach is adopted for design of the CBM. It requires the f-d curves of the CBMs to evaluate the objective functions. Finite element method is utilized to obtain the nonlinear f-d curves of the CBMs due to the geometry complexity. Chen and Ma [31] developed an analytical model to obtain f-d curves of three-segmented bistable mechanisms. Their model could be useful for modeling the CBM with five-segmented beams.

5. Conclusions

A crab-like CBM is designed and demonstrated for nearly equal switching forces in forward and backward directions. The five-section beam design increased the design degree of freedom to achieve the design objectives. The lateral stiffness of the CBM is decreased due to the stepped, slanted beam section and, therefore, the distance between the first and second stable equilibrium positions can be shortened. Given the design flexibility of the CBM, desired f-d characteristics can be obtained. The difference of the switching forces between the forward and backward motion is nearly 1.4% based on the
experiments. Such a mechanism has potential applications in nonvolatile memory devices and the threshold accelerometers with two sensing directions along a single sensing axis.

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