A Nonrational B-Spline Profiled Horn with High Displacement Amplification for Ultrasonic Welding

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June 4, 2014

Abstract

A new horn with high displacement amplification for ultrasonic welding is developed. The profile of the horn is a nonrational B-spline curve with an open uniform knot vector. The ultrasonic actuation of the horn exploits the first longitudinal displacement mode of the horn. The horn is designed by an optimization scheme and finite element analyses. Performances of the proposed horn have been evaluated by experiments. The displacement amplification of the proposed horn is 41.4% and 8.6% higher than that of the traditional catenoidal horn and a Bézier-profile horn, respectively, with the same length and end surface diameters. The developed horn has a lower displacement amplification than the nonuniform rational B-spline profiled horn but a much smoother stress distribution. The developed horn, the catenoidal horn, and the Bézier horn are fabricated and used for ultrasonic welding of lap-shear specimens. The bonding strength of the joints welded by the OUNBS horn is the highest among the three horns for the various welding parameters considered. The locations of the failure mode and the distribution of the voids of the specimens are investigated to explain the reason of the high bonding strength achieved by the OUNBS horn.

PACS: 87.55.de; 06.30.Bp
Keywords: B-spline horn; Displacement amplification; Ultrasonic welding

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1. Introduction

Ultrasonic plastic welding has been extensively used for joining thermoplastic parts in automotive, electronic appliances, and medical equipments industries. The ultrasonic welding is a solid state process, where parts are welded at the faying surfaces by high energy vibratory motion of ultrasonic horns while the work pieces are held together under static pressure [1]. During the welding process, heat is generated at the joint surface and the thermoplastic material is melted, then cooled to form a strong bond.

Successful applications of ultrasonic plastic welding require high displacement amplification of ultrasonic horns in order to melt the thermoplastics [2]. Typical profiles of ultrasonic horns are Gaussian [3], Fourier [4], exponential [5], stepped [6,7], sinusoidal [8], conical and catenoidal [9]. In comparing with the commonly used conical, exponential, catenoidal, and stepped horns, new horns with parametric curve profiles may offer higher displacement amplification [10]. Parametric curves give designers a much better control over the profile of horns to find higher displacement amplification while keeping the stress in the horns low. Wang et al. [10] designed a Bézier-profile horn whose displacement amplification is 71% higher than that of the traditional catenoidal horn with the same length and end surface diameters.

From a mathematical point of view, the flexibility of Bézier curve is limited by its two characteristics [11]. First, the number of polygon vertices fixes the order of the polynomial which defines the curve. Second, a local change within a curve is inhibited due to the global nature of the Bernstein basis. Therefore, a parametric curve, such as B-spline curve, defined by a nonglobal basis which allows the degree of the curve to be
changed without changing the number of the defining polygon vertices can be used to
describe the profile of horns for higher displacement amplification.

In this paper, we develop a horn with a nonrational B-spline profile for high
displacement amplification with applications in ultrasonic welding. The developed horn
is similar to the Bézier horn proposed by Wang et al [10], but with a different parametric
curve as the horn’s profile. The optimal designs of the horns are sought by a
multiobjective optimization algorithm. Displacements and stress distributions of various
horns, including a nonrational B-spline horn, a rational B-spline horn, a Bézier horn, a
catenoidal horn, and a stepped horn, are examined by finite element analyses. Prototypes
of the nonrational B-spline horn, the Bézier horn and the catenoidal horn are fabricated
by a numerical control (NC) machining process. Bonding strength of the ultrasonic
welded joints in plastic lap-shear specimens is investigated by experiments. The
nonrational B-spline horn, the Bézier horn and the catenoidal horn were used in the
experiments. Performance of the developed horn is compared with the traditional
catenoidal horn and the Bézier horn.

2. Horn design

It is critical to select an appropriate horn for ultrasonic welding in order to supply
sufficient power and displacement amplitude at the welded joint of the work pieces. Fig.
1(a) and (b) shows solid models of the commonly used stepped horn and catenoidal horn,
respectively. The displacement amplification of catenoidal horns is greater than that of
exponential or conical ones and less than that of stepped horns [12]. The high stress
occurring near the abruptly changing section of stepped horns renders it unfavorable. Fig.
1(c) shows a solid model of a Bézier horn. Based on a study carried out by Wang et al. [10], the Bézier horn has larger displacement amplification than the catenoidal horn, and lower stress concentration than the stepped horn. Using parametric curves as the profiles of the horns, higher displacement amplification can be achieved while keeping the stress in the horns low. In this investigation, an open uniform nonrational B-spline (OUNBS) horn, see Fig. 1(d), is proposed for ultrasonic plastic welding for its superior displacement amplification. Here, another parameter curve of an nonuniform rational B-spline (NURBS) horn as shown in Fig. 1(e) is also studied for comparison.

The design of the OUNBS horn is based on an optimization procedure where the profile of the horn is optimized via the parameters of an OUNBS curve to meet the requirement of displacement amplification. The OUNBS curve is determined by a five-point polygon \( Q_1Q_2Q_3Q_4Q_5 \) as shown in Fig. 2. The nonrational B-spline curves are included as a special case of rational B-spline curves. The parametric rational B-spline curve is given by [11]

\[
P(t) = \frac{\sum_{i=1}^{n+1} P_{i} h_{i} N_{i,k} (t)}{\sum_{i=1}^{n+1} h_{i} N_{i,k} (t)}
\]

where \( t \) is the parameter, \( P_{Q_i} \) the position vector of the point \( Q_i \). The homogeneous coordinates \( h_i \) are weights of their corresponding polygon vertex, \( P_{Q_i} \), and \( N_{i,k} \) the normalized B-spline base functions. The \( i \)th normalized B-spline basis function of order \( k \) (degree \( k - 1 \)), \( N_{i,k} (t) \), are defined as [11]

\[
N_{i,k} (t) = \begin{cases} 
1 & \text{if } x_i \leq t < x_{i+1} \\
0 & \text{otherwise}
\end{cases}
\]
and

\[ N_{i,k}(t) = \frac{(t-x_i)N_{i,k-1}(t)}{x_{i+k-1} - x_i} + \frac{(x_{i+k} - t)N_{i+k+1,k-1}(t)}{x_{i+k} - x_{i+1}} \]  (3)

The values of \( x_i \) are elements of a knot vector satisfying the relation \( x_i \leq x_{i+1} \). The parameter \( t \) varies from \( t_{\text{min}} \) and \( t_{\text{max}} \), where the values of \( t_{\text{min}} \) and \( t_{\text{max}} \) are those of the smallest element and largest element in the knot vector, respectively. The open knot vector \( X_o \) is given by [11]

\[
\begin{align*}
  x_i &= 0 & 1 \leq i \leq k \\
  x_i &= i - k & k + 1 \leq i \leq n + 1 \\
  x_i &= n - k + 2 & n + 2 \leq i \leq n + k + 1
\end{align*}
\]  (4)

A nonrational B-spline curve is yielded with all \( h_i = 1 \) in Eq. (1). The profile of the horn is optimized by allowing points \( Q_2, Q_3 \) and \( Q_4 \) to move in the design space enclosed by the dashed rectangle in Fig. 2. The positions of the points \( Q_i \) and \( Q_s \) are fixed by the specified radius of the back and front end of the horn, \( R_1 \) and \( R_2 \), respectively, and the length of the horn, \( L \). The horn is assumed to be axisymmetric.

The design of the horn follows an optimization procedure developed by Wang et al. [10]. The nondominated sorting genetic algorithm [13] is applied to the optimization of the horn profile. The algorithm is suitable for solving constrained multiobjective problems.

In the optimization process, initially, the working frequency \( f \) and the geometry parameters \( D_1, D_2 \) and \( L \) are specified. Fig. 3(a) shows the geometry parameters of the OUNBS horn. The horn has a length \( L \) and diameters \( D_1 \) and \( D_2 \) of its back and front ends, respectively. The dimensions of a Langevin transducer to drive the horn are also indicated in the figure. The objective functions of the optimization problem are
\[
\begin{align*}
\text{Min} & \quad |f_0 - f| \\
\text{Max} & \quad M = \left| \frac{u_{01}}{u_{02}} \right|
\end{align*}
\]

where \( f_0 \) is the first longitudinal modal frequency of the population of each generation of the horn. \( M \) is the amplification of the displacement defined by the ratio of the longitudinal displacement at the front end to that of the back end of the horn.

The proposed horn is designed to have the same working frequency as the Langevin transducer. Due to the geometry complexity, the modal frequency \( f_0 \) and the displacement amplification \( M \) of the OUNBS horn cannot be calculated analytically. Finite element analysis (FEA) by a commercial software ANSYS is utilized to obtain \( f_0 \) and \( M \) of the horn.

3. Analyses

3.1 Finite element model

Fig. 3(b) shows a mesh for a three-dimensional finite element model. The finite element model has 62964 8-node elements. A Cartesian coordinate system is shown in Fig. 3(a). The displacements in the \( x \), \( y \) and \( z \) directions of the nodes on the circumference of the nodal flange are constrained. A voltage is applied to the piezoelectric disk for harmonic analyses of the Langevin transducer and the horn.

In this investigation, the piezoelectric disk, front and back metal blocks of the Langevin transducer, and the horn are assumed to be linear elastic materials. A lead zirconate titanate material (PZT-5H), an aluminum alloy (AA 7075), and mild steel (SS 41) are used for the piezoelectric disk, the metal blocks of the Langevin transducer, and
the horn material, respectively. Their material properties are listed in Table 1. The commercial finite element program ANSYS is employed to perform the computations.

3.2 Numerical analysis

In the optimization process, the number of generations, N, is set to be 150, and the population of each generation is taken as 20. The knot vector $X = \begin{bmatrix} 0 & 0 & 0 & 1 & 2 & 2 & 2 \end{bmatrix}$, is determined by Eq. (4). The length $L$ and the diameters $D_1$ and $D_2$ of the back and front end of the horn are specified as 95 mm, 20 mm and 5 mm, respectively. The working frequency $f$ is set to be 28.0 kHz. In the experiments, the fabricated horn is driven by a Langevin transducer. The available commercial Langevin transducer is purchased from a local vendor. The working frequency of the Langevin transducer is a known and fixed parameter. A modal analysis of each population is performed in order to find its first longitudinal modal frequency $f_0$ and displacement amplification $M$. The horn is designed to have its first longitudinal modal frequency equal to the working frequency of the transducer.

Fig. 4 shows the profile of an optimized OUNBS horn. The positions of its control points are also shown in the figure. In order to compare the performances of the proposed horn with classical horns and other parameter curve profiled horns, a catenoidal horn, a stepped horn, a 5-point Bézier horn, and a 5-point NURBS horn are also modeled. The homogeneous coordinates $h_i$ of the NURBS horn are $[1 \ 15 \ 1 \ 15 \ 1]$. Their profiles are seen in Fig. 4. The nonuniform knot vector $X_n$ of a NURBS curve is given by [11]
where \( c_i = |P_{\theta,i} - P_{\theta}| \). The weights \( h_i \) provide additional blending capability that as \( h_i \) increases the curve is pulled closer to \( P_{\theta} \).

For fair comparison, the catenoidal horn, the 5-point Bézier horn, and the 5-point NURBS horn have the same back and front end radiuses and length as those of the proposed horn. The working frequency of the catenoidal horn, the 5-point Bézier horn, and the 5-point NURBS horn obtained by finite element analyses are 27.84 kHz, 28.00 kHz, and 28.09 kHz, respectively. The back and front end radiuses of the stepped horn are the same as those of the proposed horn. In order to have the same working frequency of 28.0 kHz as the proposed horn, the length of the stepped horn can be calculated analytically by assuming the length of both sections equal to a quarter of the ultrasonic wavelength of the material. The calculated length of the stepped horn is 89 mm. As shown in Fig. 4, the proposed OUNBS horn and the Bézier horn have bell shaped profiles. The profile of the NURBS is similar to that of the stepped horn.

Fig. 5(a) is a plot of the normalized displacements along the normalized length of the horns based on finite element computations. The displacements are normalized by the displacement at the back end of the horns. Therefore, the normalized displacement at the normalized length of 1 represents the displacement amplification. The stepped horn has the largest displacement amplification among the five types of the horns, i.e. the catenoidal horn, the stepped horn, the 5-point Bézier horn, the 5-point OUNBS horn and
the 5-point NURBS horn. The NURBS horn’s displacement amplification is slightly lower than that of the stepped horn. The new horn developed in this investigation, OUNBS horn, has larger displacement amplification than the Bézier horn and the catenoidal horn. The values of the displacement amplification of the OUNBS horn and the Bézier horn are more than twice of that of the catenoidal horn.

Fig. 5(b) shows the von Mises stress along the normalized length of the horns based on the finite element computations. For the stepped horn and the NURBS horn, high stress occurs near the abruptly changing section. Stress concentration of the OUNBS horn and the Bézier horn is significantly less than that of the stepped horn and the NURBS horn. The lower Von Mises stress of the proposed horn and the Bézier horn can be attributed to their bell-shaped profiles. The values of the displacement amplification $M$ and maximum von Mises stress $\sigma_{\text{max}}$ of the five types of the horns are listed in Table 2. Although the stepped horn and the NURBS horn give high displacement amplification, their high stress concentration near the abruptly changing section makes them prone to failure. The new horn discussed in this paper has larger displacement amplification than the Bézier horn and the catenoidal horn, and much lower stress concentration than the NURBS horn and the stepped horn. The commonly used catenoidal horn with the lowest stress among the horns considered may be regarded as inferior due to its small displacement amplification. The high stresses occurring near the abruptly changing section of the NURBS horn and the stepped horn are not favored. The developed OUNBS horn has a high potential in power ultrasonic applications, such as ultrasonic welding, based on the criteria of displacement amplification and stress distribution.
4. Fabrication and experiments

In order to verify the effectiveness of the proposed horn, prototypes of an OUNBS horn, a Bézier horn and a catenoidal horn were fabricated by numerical control machining from a stainless steel (SS 41). Dimensions of the prototypes are based on the finite element analyses. Fig. 6 is a photo of the fabricated OUNBS horn, Bézier horn, and catenoidal horn. The NURBS horn and stepped horn are not fabricated for comparison due to their stress concentration, which renders them prone to failure. Fig. 7 schematically shows a lap-shear specimen used to investigate the bonding strength of ultrasonically welded joints. The material of the specimens is polycarbonate, which has a glass transition temperature of about 147 °C, so it softens gradually above this point. The melting temperature of the material is nearly 155 °C. The weld nugget is idealized as a cylinder. The lap-shear specimen has a thickness of 1 mm, a width of 25 mm, an indentation diameter of approximately 12 mm, an overlap length of the upper and lower sheets being 50 mm, and a length of 100 mm. The dimensions of the specimen and testing procedure suggested by ASTM D3163-73, “Standard Test Method for Determining the Strength of Adhesively Bonded Rigid Plastic Lap-Shear Joints in Shear by Tension Loading”, were generally employed. As also shown in Fig. 7, two spacers with a length of 30 mm are attached to the both ends of the lap-shear specimen to induce a pure shear to the interfacial plane of the nugget for the two sheets and to avoid the initial realignment during testing. The indentation on the surface of the upper sheet of the specimen is caused by the tool plunging into the upper sheet of the specimen.
Fig. 8 is a schematic of the experimental apparatus for ultrasonically welding of the lap shear specimen. The horns were clamped and mounted on an optical table. AC voltages were applied to the Langevin transducer by an electronic circuit that works like a nearly ideal voltage source. Polycarbonate specimens were clamped onto an anvil and attached to a horizontal slider, which is free to travel along the guide. A cable connected to a weight was fastened to the anvil in order to investigate the effects of the applied static load. A load cell (LTZ-50kA, Kyowa Electronic Instruments Co., Ltd.) was placed behind the anvil to measure the static force during ultrasonic welding. We used a data acquisition unit (DBU-120A, Kyowa Electronic Instruments Co., Ltd.) with a sampling rate up to $20 \times 10^3$ samples/sec to acquire signals from the load cell. The vibration amplitude of the horns was measured by a laser head LK-H020 and a laser displacement sensor (LK-G5001, KEYENCE Corporation). The displacement measurement was recorded and analyzed by a KEYENCE software (LK-Navigator2).

For the conventional ultrasonic welding process, the important processing parameters are the welding time and the static load. As shown in Fig. 8, during the ultrasonic welding process, the static load is controlled by the weight. The static loads applied in this investigation were 700 g, 750 g, and 800 g. The welding time ranging from 1 sec to 3.5 sec was considered. The parameters were selected after trials. The lap-shear specimens were then tested by using a tensile testing machine (MTS Criterion Model 43, MTS Systems Co., US) at a monotonic displacement rate of 1.27 mm per minute. The load and displacement were simultaneously recorded during the test. Tests were terminated when the two sheets of the specimen were separated. The bonding strength is taken as the maximum load that of the lap-shear specimen can withstand while
being pulled before failing. The experimental procedure is similar to the work done by Roopa Rani and Rudramoorthy [1].

Fig. 9(a-c) shows the averaged bonding strength of the lap-shear specimens as a function of the welding time with bonding load of 700 g, 750 g and 800 g, respectively. The experiments were repeated three times for each set of the welding parameters. The average value of the three tests was recorded. Under the applied loads of 700 g, 750 g and 800 g, the joints welded by the OUNBS horn have the maximum bonding strengths of 261 N, 287 N and 289 N, respectively, and the optimum welding times are 2 sec, 1.5 sec and 1.5 sec, respectively. The high bonding strength of the joints welded by OUNBS horn can be attributed to the high temperature at the interface between the upper and lower sheets of the specimen, which results in better melting and bonding [1]. As shown in Fig. 9(a-c), when the welding time increases, the bonding strength of the specimens generally increases initially, reaches its maximum, then decreases, except the specimens welded by the catenoidal horn under the bonding load of 800 g, where its bonding strength increases initially, then shows a plateau over 2.5 sec of welding time for the cases tested.

Fig. 10(a-c) shows the infrared images of the specimens and the OUNBS horn, Bézier horn and catenoidal horn, respectively, during ultrasonic welding of 1.5 sec welding time and 800 g bonding load. Fig. 10(d-f) are the photos of the entire specimens and the horns corresponding to Fig. 10(a-c). The maximum temperatures at the surface of the specimens welded by the OUNBS horn, Bézier horn and catenoidal horn are 268.3 °C, 183.2 °C and 58.8 °C, respectively. The actual maximum temperature should be higher and may occur within the specimen. The maximum temperatures of the specimens
welded by the OUNBS horn and Bézier horn are well over the melting temperature of the material, 155°C, which caused the material to melt and flow and therefore high bonding strength upon solidification. The high bonding strength can be attributed to the glass transition of polymeric material. There is a sharp increase in the stiffness of an polymer when its temperature is reduced below the glass transition temperature.

The high temperature of the specimen during ultrasonic welding is due to the high amplitude/energy of the ultrasonic horn [1]. Fig. 11 shows the measured vibration amplitude of the catenoidal horn, the Bézier horn and the OUNBS horn as functions of the applied voltage. The catenoidal horn, the Bézier horn and the OUNBS horn are driven at their resonant frequencies, which are 27.90 kHz, 28.11 kHz and 28.06 kHz, respectively. The vibration amplitudes of the horns increase as the driving voltage increases. The amplitude of the OUNBS horn is approximately 8.6% and 41.4% greater than those of Bézier horn and the catenoidal horn, respectively, for the driving voltages considered. It is the large amplitude of the OUNBS horn provides the high energy to heat the specimens during ultrasonic welding, and to cause better melting and bonding of the thermoplastic material.

Micrographs were obtained in order to evaluate the weld quality of specimens welded by various horns. Specimens with a welding time of 1.5 sec and a bonding load of 800 g were sectioned at the mid-sections of the welds. The specimens were cross sectioned, mounted and polished to better visualize the weld nugget and base material. Fig. 12(a-c) shows the micrographs of the cross section of specimens welded by OUNBS horn, Bézier horn and catenoidal horn, respectively. Fig. 12(a) shows that the weld nugget of the specimen welded by OUNBS horn extends past the indentation zone of
diameter 5 mm, i.e. the diameter of the front end of the horn. The weld nuggets of the specimens welded by the Bézier horn and catenoidal horn are smaller than that welded by the OUNBS horn. As seen in Fig. 12(a), the weld nugget appears to be well stirred and melted to form a high quality of bonding. The relatively large size of the voids can be attributed to the large vibration amplitude of the OUNBS horn to cause more air trapped during the welding. The heat of welding may cause the trapped air to expand. The sizes of the voids in the weld of the specimens welded by the Bézier horn and catenoidal horn appear to be smaller than those of the OUNBS horn. The highly distorted surfaces near the indentation zone of the specimen welded by OUNBS horn compared to the less distorted surfaces near the indentation zone of the specimens welded by the Bézier horn and catenoidal horn may indicate that the specimens welded by the OUNBS horn may experience larger stir motion, higher plastic deformation, larger amount of melting and solidification than those welded by he Bézier horn and catenoidal horn.

Fig. 12(d-f) shows the micrographs of the cross section of the failed specimens welded by OUNBS horn, Bézier horn and catenoidal horn, respectively. The welded specimens exhibited two competing failure locations. The specimens welded by the OUNBS horn failed near and outside the weld nugget, see the cross sections of the failed specimens in Fig. 12(d). The failed location seems to suggest that the material near the weld nugget has a much higher strength than the base material. The specimen welded by Bézier horn failed closer to the weld nugget than that welded by the OUNBS horn, see the cross sections of the failed specimens in Fig. 12(e). The location of the crack of the failed specimen suggests that the material near this location is relatively weak and the necking failure occurred near this region. The specimens welded by the catenoidal horn
failed outside the weld nugget, see Fig. 12(f). The cracks of failure appear to initiate near the circumference of the bonding area of the specimen and grow along the thickness direction of the specimen. The material near the weld nugget welded by the catenoidal horn did not undergo a glass transition, therefore, there is no sharp increase in the stiffness of the material as the specimens welded by the OUNBS horn and the Bézier horn.

Fig. 12(a-c) also shows the microhardness distribution on the cross section of specimens welded by OUNBS horn, Bézier horn and catenoidal horn, respectively. The material near the weld nugget has an increase in microhardness compared to the base material for the three cases. The location with the highest value of hardness is near the center of the joints. It can be seen that the specimen welded by the OUNBS horn has the largest heat affected zone, where the hardness is higher than that of the base material, followed by the specimen welded by the Bézier horn. The specimen welded by the catenoidal horn has the smallest heat affected zone. The results shown in Fig. 12 seem to suggest that an increase in microhardness may produce an increase in bonding strength of the specimens. The main advantage of using the OUNBS horn for ultrasonic welding can be the formation of larger weld nuggets, and results in joints with superior bonding strength compared to the Bézier horn and the catenoidal horn. The presence of larger voids in the nugget of the specimen welded by the OUNBS horn can be taken as minor defects related to thermal shrinkage, entrapped air or physical-chemical structural changes.
5. Conclusions

A new horn with high displacement amplification for ultrasonic welding is proposed. The profile of the horn is a nonrational B-spline curve with an open uniform knot vector. The OUNBS horn is designed by using a multiobjective optimization algorithm and finite element analyses to optimize its displacement amplification. Based on the finite element analyses, the stepped horn and the NURBS horn may give high displacement amplifications, but their high stress concentration near the abruptly changing section makes them prone to failure. The developed OUNBS horn has larger displacement amplification than the Bézier horn and the commonly used catenoidal horn, and much lower stress concentration than the NURBS horn and the stepped horn. Prototypes of the horn have been fabricated and tested to verify the effectiveness of the proposed new horn. The displacement amplification of the proposed horn is 41.4% and 8.6% higher than that of the traditional catenoidal horn and a Bézier horn, respectively, with the same length and end surface diameters.

Lap-shear specimens are ultrasonically welded by the fabricated catenoidal horn, Bézier horn and OUNBS horn. The joints of the specimens welded by the OUNBS horn have the maximum bonding strengths for the various welding parameters considered. The experimental results verify the effectiveness of the developed OUNBS horn. The proposed horn may be more suitable than the classical stepped and catenoidal horns and other parametric curve profiled horns in application where high displacement amplification and low stress concentration are required.
Acknowledgement

Support of this work by a grant from National Science Council, Taiwan, ROC (Grant Number: NSC 102-2221-E-005-069) is greatly appreciated. The authors would like to express their appreciation to the National Center for High-Performance Computing (NCHC), Taiwan for their assistance. Helpful discussions with Dr. Jun-Yen Uan with the Department of Material Science and Engineering, National Chung Hsing University, Taiwan, ROC, are greatly appreciated.
References


**Biography**

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Table 1. Properties of the piezoelectric disk (PZT-5H), metal blocks of the Langevin transducer (AA 7075) and the horn (SS 41)

<table>
<thead>
<tr>
<th>Property</th>
<th>Tensor (in order of x, y, z, xy, xz, yz)</th>
<th>PZT-5H Piezoelectricity (C m(^{-2}))</th>
<th>Permittivity (F m(^{-1}))</th>
<th>Stiffness (N m(^{-2}))</th>
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<tr>
<td></td>
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<td>[0, -6.55, 0, 0, 0, 0]</td>
<td>(10^{-9} \times \begin{bmatrix} 15.052 &amp; 0 &amp; 0 \ 0 &amp; 13.015 &amp; 0 \ 0 &amp; 0 &amp; 15.052 \end{bmatrix} )</td>
<td>(10^{10} \times \begin{bmatrix} 12.72 &amp; 8.47 &amp; 8.02 &amp; 0 &amp; 0 &amp; 0 \ 11.74 &amp; 8.47 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 12.72 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \end{bmatrix} )</td>
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<td>Density (kg m(^{-3})) 7600</td>
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<td>AA 7075</td>
<td>Young’s modulus (GPa) 69</td>
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<td>Poisson’s ratio 0.35</td>
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<td></td>
<td>Density (kg m(^{-3})) 2730</td>
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<td>SS 41</td>
<td>Young’s modulus (GPa) 210</td>
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<td>Poisson’s ratio 0.3</td>
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<td>Density (kg m(^{-3})) 7800</td>
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Table 2. Comparison of the five types of the horns

<table>
<thead>
<tr>
<th>Type</th>
<th>( R_1 / R_2 )</th>
<th>Length (mm)</th>
<th>( M )</th>
<th>( \sigma_{\text{max}} ) (MPa)</th>
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</thead>
<tbody>
<tr>
<td>Catenoidal</td>
<td>4</td>
<td>95</td>
<td>5.1</td>
<td>71</td>
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<tr>
<td>Stepped</td>
<td>4</td>
<td>89</td>
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