Microwave Amplifier Design
• **Two-Port Power Gains**

\[
\begin{align*}
    \text{Power Gain} &= G = \frac{P_L}{P_{in}} ([S], \Gamma_L) \\
    \text{Available Gain} &= G_A = \frac{P_{avn}}{P_{avs}} ([S], \Gamma_S) \\
    \text{Transducer Power Gain} &= G_T = \frac{P_L}{P_{avs}} ([S], \Gamma_S, \Gamma_L)
\end{align*}
\]

**Figure 11-1 (p. 537)**  A two-port network with general source and load impedances.

\[
\begin{align*}
    P_{avs} &= P_{in} \bigg|_{\Gamma_{in}=\Gamma_S^*} \\
    P_{avn} &= P_L \bigg|_{\Gamma_L=\Gamma_{out}^*} \\
    \Gamma_L &= \frac{Z_L - Z_0}{Z_L + Z_0} \\
    \Gamma_S &= \frac{Z_S - Z_0}{Z_S + Z_0}
\end{align*}
\]
Two Port Power Gains

\[ P_{in} = \frac{|V_1^+|^2}{2Z_0} \left(1 - |\Gamma_{in}|^2 \right) = \frac{|V_S|^2}{8Z_0} \frac{|1-\Gamma_S|^2}{|1-\Gamma_S\Gamma_{in}|^2} \left(1 - |\Gamma_{in}|^2 \right) \]

\[ V_2^- = S_{21}V_1^+ + S_{22}V_2^+ = S_{21}V_1^+ + S_{22}\Gamma_LV_2^- \]

\[ P_L = \frac{|V_2^-|^2}{2Z_0} \left(1 - |\Gamma_{L}|^2 \right) = \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2|1-\Gamma_S\Gamma_{in}|^2} \left(1 - |\Gamma_{L}|^2 \right) \]

\[ G = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{\left(1 - |\Gamma_{in}|^2 \right) |1-S_{22}\Gamma_L|^2}. \]
Two Port Power Gains

\[
P_{\text{avs}} = P_{\text{in}} \bigg|_{\Gamma_{\text{in}} = \Gamma_{\text{s}}^*} = \frac{|V_S|^2 \left| 1 - \Gamma_S \right|^2}{8Z_0 \left( 1 - |\Gamma_S|^2 \right)}
\]

\[
P_{\text{avn}} = P_L \bigg|_{\Gamma_{L} = \Gamma_{\text{out}}^*} = \frac{|V_S|^2 \left| S_{21} \right|^2 \left( 1 - \left| \Gamma_{\text{out}} \right|^2 \right) \left| 1 - \Gamma_S \right|^2}{8Z_0 \left| 1 - S_{22} \Gamma_{\text{out}}^* \right|^2 \left| 1 - \Gamma_S \Gamma_{\text{in}} \right|^2}
\]

\[
P_{\text{av}} = P_{\text{in}} \bigg|_{\Gamma_{\text{in}} = \Gamma_{\text{out}}^*} = \frac{|V_S|^2 \left| S_{21} \right|^2 \left| 1 - \Gamma_S \right|^2}{8Z_0 \left( 1 - S_{11} \Gamma_S \right)^2 \left( 1 - \left| \Gamma_{\text{out}} \right|^2 \right)}
\]

\[
G_A = \frac{P_{\text{avn}}}{P_{\text{avs}}} = \frac{|S_{21}|^2 \left( 1 - \left| \Gamma_S \right|^2 \right)}{\left| 1 - S_{11} \Gamma_S \right|^2 \left( 1 - \left| \Gamma_{\text{out}} \right|^2 \right)}
\]

\[
G_T = \frac{P_L}{P_{\text{avs}}} = \frac{|S_{21}|^2 \left( 1 - \left| \Gamma_S \right|^2 \right) \left( 1 - \left| \Gamma_L \right|^2 \right)}{\left| 1 - \Gamma_S \Gamma_{\text{in}} \right|^2 \left| 1 - S_{22} \Gamma_L \right|^2}
\]

Special case 1: both input and output matched
\[
\rightarrow \Gamma_L = \Gamma_S = 0
\]
\[
G_T = |S_{21}|^2
\]

Special case 2: unilateral transducer power gain, \( S_{12} = 0 \)
\[
\rightarrow \Gamma_{\text{in}} = S_{11}
\]
\[
G_{TU} = \frac{|S_{21}|^2 \left( 1 - \left| \Gamma_S \right|^2 \right) \left( 1 - \left| \Gamma_L \right|^2 \right)}{\left| 1 - S_{11} \Gamma_S \right|^2 \left| 1 - S_{22} \Gamma_L \right|^2}
\]
Two Port Power Gains

Example 11.1  Comparison of Power Gain Definitions

A microwave transistor has the following $S$ parameters at 10 GHz, with a 50Ω reference impedance:

$S_{11} = 0.45\angle150^\circ$, $S_{12} = 0.01\angle-10^\circ$, $S_{21} = 2.05\angle10^\circ$, $S_{22} = 0.40\angle-150^\circ$

$Z_S = 20\ \Omega$, $Z_L = 30\ \Omega$.

Solution

\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = -0.250 \]

\[ \Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = -0.429 \]

\[ \Gamma_{\text{in}} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.455\angle150^\circ \]

\[ \Gamma_{\text{out}} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} = 0.408\angle-151^\circ \]

\[ G = \frac{|S_{21}|^2\left(1 - |\Gamma_L|^2\right)}{\left(1 - |\Gamma_{\text{in}}|^2\right)|1 - S_{22}\Gamma_L|^2} = 5.94 \]

\[ G_A = \frac{|S_{21}|^2\left(1 - |\Gamma_S|^2\right)}{\left|1 - S_{11}\Gamma_S\right|^2\left(1 - |\Gamma_{\text{out}}|^2\right)} = 5.85 \]

\[ G_T = \frac{|S_{21}|^2\left(1 - |\Gamma_S|^2\right)\left(1 - |\Gamma_L|^2\right)}{|1 - \Gamma_S\Gamma_{\text{in}}|^2\left|1 - S_{22}\Gamma_L\right|^2} = 5.49 \]
Two Port Power Gains

Effective gain for the input (source) matching network:

$$G_S = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{in}\Gamma_S|^2}$$

Transducer power gain for transistor:

$$G_0 = |S_{21}|^2$$

Effective gain for the output (load) matching network:

$$G_L = \frac{1-|\Gamma_L|^2}{|1-\Gamma_{22}\Gamma_L|^2}$$

The overall transducer gain:

$$G_T = G_S G_0 G_L$$

**Figure 11-2 (p. 540)** The general transistor amplifier circuit.
Two Port Power Gains

- If the transistor is unilateral, so that \( S_{12} = 0 \)
  \[ \Gamma_{\text{in}} = S_{11}, \quad \Gamma_{\text{out}} = S_{22} \]
  \[ G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \Gamma_S|^2} \]
  \[ G_0 = |S_{21}|^2 \]
  \[ G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \Gamma_L|^2} \]
  The overall transducer gain: \( G_T = G_S G_0 G_L \)

Conjugate match GaAs FET equivalent CKT

\[ X = \frac{1}{\omega C_{gs}} \rightarrow Z_{\text{in}} = Z_S^* \]

\[ B = -\omega C_{ds} \rightarrow Z_{\text{out}} = Z_L^* \]

\[ V_c = \frac{V_S}{2 j \omega R_t C_{gs}} \]

\[ G_{TU} = \frac{P_L}{P_{\text{avs}}} = \frac{1}{8} \left| g_m V_c \right|^2 \frac{R_{ds}}{R_i} = \frac{g_m^2 R_{ds}}{4 \omega^2 R_t C_{gs}^2} = \frac{R_{ds}}{4 R_i} \left( \frac{f_T}{f} \right)^2 \]
Figure 11-4 (p. 542)
Photograph of a low noise MMIC amplifier using three HEMTs with coplanar waveguide circuitry. The amplifier has a gain of 20 dB from 20 to 24 GHz. The contact pads on the left and right of the chip are for RF input and output, with DC bias connections at the top. Chip dimensions are $1.1 \times 2.0$ mm. Courtesy of R. W. Jackson and B. Hou of the University of Massachusetts and J. Wendler of M/A-COM.
Stability

- Unconditional stability:
  \[ |\Gamma_{in}| < 1, \quad |\Gamma_{out}| < 1 \]
  for all passive source and load impedances
  i.e., \[ |\Gamma_S| < 1, \quad |\Gamma_L| < 1 \]

- Conditional stability:
  \[ |\Gamma_{in}| < 1, \quad |\Gamma_{out}| < 1 \]
  only for a certain range of passive source and load impedances,
  This case is also referred to as potentially unstable

- Frequency dependence

- Rigorous treatment of stability requires that the network S parameters have
  no poles in the right-half complex frequency plane, in addition to
  \[ |\Gamma_{in}| < 1 \]
  and \[ |\Gamma_{out}| < 1 \].
• **Stability Circles**
Conditions for unconditionally stable amplifier:

\[
|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1
\]

\[
|\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \right| < 1
\]

If the device is unilateral \((S_{12} = 0)\) these conditions reduce to:

\[
|S_{11}| < 1
\]

\[
|S_{22}| < 1
\]

The equation for the output stability circle:

\[
|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| = 1
\]

or \(S_{11}(1 - S_{22}\Gamma_L) + S_{12}S_{21}\Gamma_L = |1 - S_{22}\Gamma_L|\)

\[
\Delta = S_{11}S_{22} - S_{12}S_{21}
\]

\[
\rightarrow |S_{11} - \Delta \Gamma_L| = |1 - S_{22}\Gamma_L|
\]

\[
\Gamma_L \Gamma_L^* = \frac{(S_{22} - \Delta S_{11}^*)\Gamma_L + (S_{22}^* - \Delta^* S_{11})\Gamma_L^*}{|S_{22}|^2 - |\Delta|^2} = \frac{|S_{11}|^2 - 1}{|S_{22}|^2 - |\Delta|^2}
\]

Adding \(|S_{22} - \Delta S_{11}^*|^2 / (|S_{22}|^2 - |\Delta|^2)^2\) to both sides

\[
\rightarrow \left| \Gamma_L - \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \right| = \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2}
\]
Stabili
ty

• Stability Circles
Output stability circle:
\[
|\Gamma_L - C_L| = R_L
\]
\[
C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}
\text{ (center)}
\]
\[
R_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right|
\text{ (radius)}
\]

If we set \( Z_L = Z_0 \), then \( \Gamma_L = 0 \)
\( \rightarrow |\Gamma_{in}| = |S_{11}| \)
\( \Rightarrow |S_{11}| < 1 \Rightarrow \text{Figure 11.5(a)} \)
\( \Rightarrow |S_{11}| > 1 \Rightarrow \text{Figure 11.5(b)} \)

Input stability circle:
\[
|\Gamma_S - C_S| = R_S
\]
\[
C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2}
\text{ (center)}
\]
\[
R_S = \left| \frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2} \right|
\text{ (radius)}
\]

If the device is unconditional stable,
\( \rightarrow \) stability circle is completely
outside the Smith chart.
\[
|C_L - R_L| > 1, \text{ for } |S_{11}| < 1
\]
\[
|C_S - R_S| > 1, \text{ for } |S_{22}| < 1
\]
Stability

Figure 11-5 (p. 544)
Output stability circles for a conditionally stable device. (a) $|S_{11}| < 1$. (b) $|S_{11}| > 1$. 
• Tests for Unconditional Stability

$K - \Delta$ test:

Rollet's condition

$$K \equiv \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1,$$

Auxiliary condition

$$|\Delta| \equiv |S_{11}S_{22} - S_{12}S_{21}| < 1,$$

are simultaneously satisfied.

Also, we must have

$$|S_{11}| < 1$$

$$|S_{22}| < 1$$

$\mu$-test:

$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \right| = \frac{|S_{22} - \Delta\Gamma_S|}{1 - S_{11}\Gamma_S} < 1$$

$\Gamma_S$ boundary

$$|\Gamma_{out}| = |e^{i\phi}| = \left| \frac{S_{22} - \Gamma_{out}}{\Delta - S_{11}\Gamma_{out}} \right| = 1$$

$$\Rightarrow \left| \Gamma_{out} + \frac{\Delta S^*_{11} - S_{22}}{1 - |S_{11}|^2} \right|^2 = \frac{|S_{12}S_{21}|^2}{(1 - |S_{11}|^2)^2}$$

$$\Leftrightarrow |\Gamma_{out} - C| = R$$

$$\Rightarrow |C| + R < 1 \Rightarrow |S_{22} - \Delta S^*_{11}| + |S_{12}S_{21}| < 1 - |S_{11}|^2,$$

$$\Rightarrow \mu \equiv \frac{1 - |S_{11}|^2}{|S_{22} - \Delta S^*_{11}| + |S_{12}S_{21}|} > 1.$$
Example 11.2  Transistor Stability
The $S$ parameters for the HP HFET-102 GaAs FET at 2 GHz with a bias voltage $V_{gs} = 0$ are given as follows ($Z_0 = 50 \Omega$)

$S_{11} = 0.894 \angle -60.6^\circ$,  $S_{12} = 0.02 \angle 62.4^\circ$,  $S_{21} = 3.122 \angle 123.6^\circ$,  $S_{22} = 0.781 \angle -27.6^\circ$

**Solution**

**$K - \Delta$ test** :

Rollet's condition

$$ K \equiv \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 0.607 $$

Auxiliary condition

$$ |\Delta| \equiv |S_{11}S_{22} - S_{12}S_{21}| = 0.696 $$

**$\mu - test$** :

$$ \mu \equiv \frac{1 - |S_{11}|^2}{|S_{22} - \Delta S_{11}^*| + |S_{12}S_{21}|} = 0.86 $$

$$ C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} = 1.361 \angle 47^\circ $$

$$ R_L = \frac{|S_{12}S_{21}|}{|S_{22}|^2 - |\Delta|^2} = 0.50 $$

$$ C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} = 1.132 \angle 68^\circ $$

$$ R_S = \frac{|S_{12}S_{21}|}{|S_{11}|^2 - |\Delta|^2} = 0.199 $$
Stability

Figure 11-6 (p. 549)
Stability circles for Example 11.2.
Single-Stage Transistor Amplifier Design

- Design for Maximum Gain (Conjugate Matching)

Since $G_0$ is fixed, the overall gain will be controlled by $G_s$ and $G_L$.

Maximum Gain if conjugate matching

$$\Gamma_{\text{in}} = \Gamma_s^*$$ and $$\Gamma_{\text{out}} = \Gamma_L^*$$

$$\Rightarrow G_{\text{max}} = \frac{1}{1 - |\Gamma_s|^2} \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)}{|1 - S_{22} \Gamma_L|^2}$$

In the general bilateral circuit: matching condition require

$$\Gamma_s^* = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$$

$$\Gamma_L^* = S_{22} + \frac{S_{12} S_{21} \Gamma_s}{1 - S_{11} \Gamma_s}$$

Solving for $\Gamma_s$

$$\Gamma_s = S_{11} + \frac{S_{12} S_{21} \Gamma_s}{1 - S_{22} \Gamma_L}$$

$$\Gamma_s = \frac{S_{22} - \Delta \Gamma_s}{1 - S_{11} \Gamma_s}$$

$$\Rightarrow (S_{11} - \Delta S_{22}) \Gamma_s^2 + (\Delta^2 - |S_{11}|^2 + |S_{22}|^2 - 1) \Gamma_s$$

$$+ (S_{11}^* - \Delta^* S_{22}) = 0$$

$$\gamma_s = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$$

Similarly, $\Gamma_L$
The variables $B_1$, $C_1$, $B_2$, $C_2$ are defined as

\[
B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2
\]

\[
B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2
\]

\[
C_1 = S_{11} - \Delta S_{22}^*
\]

\[
C_2 = S_{22} - \Delta S_{11}^*
\]

\[
\Rightarrow B_1^2 - 4|C_1|^2 > 0,
\]

\[
B_2^2 - 4|C_2|^2 > 0
\]

\[
\rightarrow K > 1
\]

The unilateral case:

\[
S_{12} = 0
\]

\[
\rightarrow \Gamma_s = S_{11}^*, \ \Gamma_l = S_{22}^*
\]

\[
G_{TU_{\text{max}}} = \frac{1}{1 - |S_{11}|^2} |S_{21}|^2 \frac{1}{1 - |S_{22}|^2}
\]

If the transistor is unconditionally stable, so that $K > 1$

\[
G_{T_{\text{max}}} = \frac{|S_{21}|}{|S_{12}|} \left(K - \sqrt{K^2 - 1}\right)
\]

If $K < 1$, simultaneous conjugate matching is not possible, $\rightarrow$ maximum stable gain:

\[
G_{msg} = \frac{|S_{21}|}{|S_{12}|}
\]
Single-Stage Transistor Amplifier Design

- Example 11.3  Conjugately Matched Amplifier Design

Design an amplifier for maximum gain at 4.0 GHz using single-stub matching sections. Calculate and plot the input return loss and the gain from 3 to 5 GHz. The GaAs FET has the following $S$ parameters ($Z_0 = 50 \Omega$)

<table>
<thead>
<tr>
<th>$f$ (GHz)</th>
<th>$S_{11}$</th>
<th>$S_{21}$</th>
<th>$S_{12}$</th>
<th>$S_{22}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>0.80 $\angle -89^\circ$</td>
<td>2.86 $\angle 99^\circ$</td>
<td>0.03 $\angle 56^\circ$</td>
<td>0.76 $\angle -41^\circ$</td>
</tr>
<tr>
<td>4.0</td>
<td>0.72 $\angle -116^\circ$</td>
<td>2.60 $\angle 76^\circ$</td>
<td>0.03 $\angle 57^\circ$</td>
<td>0.73 $\angle -54^\circ$</td>
</tr>
<tr>
<td>5.0</td>
<td>0.66 $\angle -142^\circ$</td>
<td>2.39 $\angle 54^\circ$</td>
<td>0.03 $\angle 62^\circ$</td>
<td>0.72 $\angle -68^\circ$</td>
</tr>
</tbody>
</table>

**Solution**

Check $K - \Delta$ test at 4.0 GHz

\[
K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 1.195
\]

\[
\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.488 \angle -162^\circ
\]

For maximum gain,

\[
\Gamma_{s} = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} = 0.872 \angle 123^\circ
\]

\[
\Gamma_{L} = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} = 0.876 \angle 61^\circ
\]
Single-Stage Transistor Amplifier Design

Solution (continued)

\[
G_S = \frac{1}{1 - |\Gamma_S|^2} = 4.17 = 6.20 \text{ dB}
\]

\[
G_0 = |S_{21}|^2 = 6.76 = 8.30 \text{ dB}
\]

\[
G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} = 1.67 = 2.22 \text{ dB}
\]

The overall transducer gain,

\[
G_{T_{\text{max}}} = 6.20 + 8.30 + 2.22 = 16.72 \text{ dB}
\]

**Figure 11-7a** (p. 552) (a) Smith chart for the design of the input matching network.

**Figure 11-7b** (p. 553) (b) RF circuit.
Figure 11-7b (p. 553)   (c) Frequency response.
• Constant Gain Circles and Design for Specified Gain

$|S_{12}|$ is small enough to ignore, → unilateral.

\[
\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2}
\]

$U$: unilateral figure of merit

\[
U = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)}
\]

\[
G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}
\]

\[
G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}
\]

\[
G_{S_{\max}} = \frac{1}{1-|S_{11}|^2}
\]

\[
G_{L_{\max}} = \frac{1}{1-|S_{22}|^2}
\]

Define normalized gain factors:

\[
g_S \equiv \frac{G_S}{G_{S_{\max}}} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} \left(1-|S_{11}|^2\right)
\]

\[
g_L \equiv \frac{G_L}{G_{L_{\max}}} = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \left(1-|S_{22}|^2\right)
\]
Single-Stage Transistor Amplifier Design

- Constant Gain Circles and Design for Specified Gain

For fixed value of $g_s$:

$$g_s |1 - S_{11} \Gamma_s|^2 = \left(1 - |\Gamma_s|^2\right)\left(1 - |S_{11}|^2\right),$$

$$\Gamma_s \Gamma_s^* - \frac{g_s \left(S_{11} \Gamma_s + S_{11}^*\right)}{1 - (1 - g_s)|S_{11}|^2} = \frac{1 - |S_{11}|^2 - g_s}{1 - (1 - g_s)|S_{11}|^2},$$

$$\left|\Gamma_s - \frac{g_s \left(S_{11}^*\right)}{1 - (1 - g_s)|S_{11}|^2}\right| = \frac{\sqrt{1 - g_s}\left(1 - |S_{11}|^2\right)}{1 - (1 - g_s)|S_{11}|^2}$$

$$\Rightarrow \left|\Gamma_s - C_s\right| = R_s$$

$$C_s = \frac{g_s S^*_{11}}{1 - (1 - g_s)|S_{11}|^2},$$

$$R_s = \frac{\sqrt{1 - g_s}\left(1 - |S_{11}|^2\right)}{1 - (1 - g_s)|S_{11}|^2}.$$
• Example 11.4 Amplifier Design for Specified Gain

Design an amplifier to have a gain of 11 dB at 4.0 GHz. Plot constant gain circles for $G_S = 2$ dB and 3 dB, and $G_L = 0$ dB and 1 dB. Calculate and plot the input return loss and overall amplifier gain from 3 to 5 GHz. The FET has the following $S$ parameters ($Z_0 = 50 \, \Omega$)

<table>
<thead>
<tr>
<th>$f$ (GHz)</th>
<th>$S_{11}$</th>
<th>$S_{21}$</th>
<th>$S_{12}$</th>
<th>$S_{22}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>0.80$\angle-90^\circ$</td>
<td>2.8$\angle100^\circ$</td>
<td>0</td>
<td>0.66$\angle-50^\circ$</td>
</tr>
<tr>
<td>4.0</td>
<td>0.75$\angle-120^\circ$</td>
<td>2.5$\angle86^\circ$</td>
<td>0</td>
<td>0.60$\angle-70^\circ$</td>
</tr>
<tr>
<td>5.0</td>
<td>0.71$\angle-140^\circ$</td>
<td>2.3$\angle60^\circ$</td>
<td>0</td>
<td>0.58$\angle-85^\circ$</td>
</tr>
</tbody>
</table>

**Solution**

$$G_{S\text{max}} = \frac{1}{1-|S_{11}|^2} = 2.29 = 3.6 \, \text{dB}$$

$$G_0 = |S_{21}|^2 = 6.25 = 8.0 \, \text{dB}$$

$$G_{TU\text{max}} = 3.6 + 1.9 + 8.0 = 13.5 \, \text{dB}$$

$$G_{L\text{max}} = \frac{1}{1-|S_{22}|^2} = 1.56 = 1.9 \, \text{dB}$$
Solution (continued)

\[ G_S = 3 \text{ dB} \quad g_S = 0.875 \]
\[ C_S = 0.706 \angle 120^\circ \quad R_S = 0.166 \]
\[ G_S = 2 \text{ dB} \quad g_S = 0.691 \]
\[ C_S = 0.627 \angle 120^\circ \quad R_S = 0.294 \]
\[ G_L = 1 \text{ dB} \quad g_L = 0.806 \]
\[ C_L = 0.520 \angle 70^\circ \quad R_L = 0.303 \]
\[ G_L = 0 \text{ dB} \quad g_L = 0.640 \]
\[ C_L = 0.440 \angle 70^\circ \quad R_L = 0.440 \]

Figure 11-8a/b (p. 556) (a) Constant gain circles. (b) RF circuit.
Single-Stage Transistor Amplifier Design

Figure 11-8b (p. 557) (c) Transducer gain and return loss.
• Low-Noise Amplifier Design

Derivation of circles of constant noise figure—

Noise figure for a two-port amplifier:

\[ F = F_{\text{min}} + \frac{R_N}{G_S} |Y_S - Y_{\text{opt}}|^2, \]

where

\[ Y_S = G_S + jB_S = \text{source admittance} \]
\[ Y_{\text{opt}} = \text{optimum source admittance} (F_{\text{min}}) \]
\[ F_{\text{min}} = \text{minimum noise figure} \]
\[ R_N = \text{equivalent noise resistance of transistor} \]
\[ G_S = \text{real part of source admittance} \]

\[ Y_S = \frac{1}{Z_0} \frac{1 - \Gamma_S}{1 + \Gamma_S} \]
\[ Y_{\text{opt}} = \frac{1}{Z_0} \frac{1 - \Gamma_{\text{opt}}}{1 + \Gamma_{\text{opt}}} \]
Single-Stage Transistor Amplifier Design

\[ |Y_S - Y_{opt}|^2 = \frac{4}{Z_0^2} \left| \Gamma_S - \Gamma_{opt} \right|^2 \]

\[ G_S = \text{Re}\{Y_S\} = \frac{1}{2Z_0} \left( \frac{1 - \Gamma_S}{1 + \Gamma_S} + \frac{1 - \Gamma_S^*}{1 + \Gamma_S^*} \right) \]

\[ = \frac{1}{Z_0} \frac{1 - |\Gamma_S|^2}{|1 + \Gamma_S|^2} \]

\[ F = F_{\min} + \frac{4R_N}{Z_0} \left( \frac{1 - |\Gamma_S|^2}{1 + |\Gamma_{opt}|^2} \right). \]

Define the noise figure parameter, \( N \):

\[ N = \frac{\left| \Gamma_S - \Gamma_{opt} \right|^2}{1 - |\Gamma_S|^2} = \frac{F - F_{\min}}{4R_N / Z_0} |1 + \Gamma_{opt}|^2 \]

\[ (\Gamma_S - \Gamma_{opt})(\Gamma_S^* - \Gamma_{opt}^*) = N \left( 1 - |\Gamma_S|^2 \right) \]

\[ \Gamma_S \Gamma_S^* - (\Gamma_S \Gamma_{opt}^* + \Gamma_S^* \Gamma_{opt}) + \Gamma_{opt} \Gamma_{opt}^* = N - N|\Gamma_S|^2 \]

\[ \Gamma_S \Gamma_S^* \left( \frac{\Gamma_S \Gamma_{opt}^* + \Gamma_S^* \Gamma_{opt}}{N + 1} \right) = \frac{N - |\Gamma_{opt}|^2}{N + 1} \]

\[ \left| \Gamma_S - \frac{\Gamma_{opt}}{N + 1} \right| = \sqrt{N \left( N + 1 - |\Gamma_{opt}|^2 \right)} \]

\[ \Rightarrow \]

\[ |\Gamma_S - C_F| = R_F \]

\[ C_F = \frac{\Gamma_{opt}}{N + 1} \]

\[ R_F = \sqrt{N \left( N + 1 - |\Gamma_{opt}|^2 \right)} \]
**Example 11.5  Low-Noise Amplifier Design**

A GaAs FET is biased for minimum noise figure, and has the following $S$ parameters and noise parameters at 4 GHz ($Z_0 = 50 \, \Omega$): $S_{11} = 0.6 \angle -60^\circ$, $S_{12} = 0.05 \angle 26^\circ$, $S_{21} = 1.9 \angle 81^\circ$, $S_{22} = 0.5 \angle -60^\circ$; $F_{\text{min}} = 1.6 \, \text{dB}$, $\Gamma_{\text{opt}} = 0.62 \angle 100^\circ$, $R_N = 20 \, \Omega$. Calculate the maximum error in $G_T$ if device is assumed unilateral. Then design an amplifier having a 2 dB noise figure with the maximum gain with this noise figure.

**Solution**

$$U = \frac{|S_{12}| |S_{21}| |S_{11}| |S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)} = 0.059$$

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2}$$

or $0.891(-0.5 \, \text{dB}) < \frac{G_T}{G_{TU}} < 1.130(0.5 \, \text{dB})$

$$N = \frac{F - F_{\text{min}}}{4R_N/Z_0} |1 + \Gamma_{\text{opt}}|^2 = 0.0986$$

$$C_F = \frac{\Gamma_{\text{opt}}}{N+1} = 0.56 \angle 100^\circ$$

$$R_F = \frac{\sqrt{N(N+1-|\Gamma_{\text{opt}}|^2)}}{N+1} = 0.24$$
Single-Stage Transistor Amplifier Design

Solution (continued)

constant gain circles

<table>
<thead>
<tr>
<th>$G_S$ (dB)</th>
<th>$g_S$</th>
<th>$C_S$</th>
<th>$R_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.805</td>
<td>0.52 $\angle 60^\circ$</td>
<td>0.300</td>
</tr>
<tr>
<td>1.5</td>
<td>0.904</td>
<td>0.56 $\angle 60^\circ$</td>
<td>0.205</td>
</tr>
<tr>
<td>1.7</td>
<td>0.946</td>
<td>0.58 $\angle 60^\circ$</td>
<td>0.150</td>
</tr>
</tbody>
</table>

$\rightarrow \Gamma_S = 0.53 \angle 75^\circ$

Choose $\Gamma_L = S_{22}^* = 0.5 \angle 60^\circ$ for maximum $G_L$:

$G_L = 1/(1 - |S_{22}|^2) = 1.33 = 1.25$ dB

$G_0 = |S_{21}|^2 = 3.61 = 5.58$ dB

$G_{TU} = G_S + G_0 + G_L = 8.53$ dB

Figure 11-9a (p. 561)

Circuit design for the transistor amplifier of Example 11.5.

(a) Constant gain and noise figure circles. (b) RF circuit.
Broadband Transistor Amplifier Design

- **Narrow bandwidth due to**
  - Matching requirement for maximum gain $\leftrightarrow$ transistors are not $50\Omega$
  - $|S_{21}|$ decreases with frequency at the rate of 6 dB/octave

- **Common approaches for broadband (cost: gain, complexity...)**
  - Compensated matching networks ($|S_{21}|$, cost: input/output matching)
  - Resistive matching networks (matching, cost: gain loss and noise figure)
  - Negative feedback (frequency response, matching, stability, cost: gain and noise figure)
  - Balanced amplifiers (broadband match, cost: twice transistor & power)
  - Distributed amplifiers (broadband gain, matching, noise figure, cost: circuit size, single stage gain)
• Balanced Amplifiers

![Diagram of a balanced amplifier using 90° hybrid couplers.](image)

**Figure 11-10 (p. 562)** A balanced amplifier using 90° hybrid couplers.

\[
\begin{pmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{pmatrix} = \frac{1}{2} \begin{pmatrix}
\Gamma_A - \Gamma_B & -j(S_{12A} + S_{12B}) \\
-j(G_A + G_B) & -(S_{22A} - S_{22B})
\end{pmatrix}
\]

IF the amplifier are identical → \( S_{11} = 0, \ G = G_A = G_B \)

\[ F = \frac{1}{2}(F_A + F_B) \]
• Example 11.6 Performance and Optimization of a Balanced Amplifier

**Figure 11-11 (p. 564)** Gain and return loss, before and after optimization, for the balanced amplifier of Example 11.6.
• Distributed Amplifiers

**Figure 11-12 (p. 565)** Configuration of an $N$-stage distributed amplifier.

**Figure 11-13 (p. 566) (a)** Transmission line circuit for the gate line of the distributed amplifier;
• Distributed Amplifiers

**Figure 11-13 (p. 566)** (b) equivalent circuit of a single unit cell of the gate line.

**Figure 11-14 (p. 566)** (b) equivalent circuit of a single unit cell of the drain line.

**Figure 11-14 (p. 566)** (a) Transmission line circuit for the drain line of the distributed amplifier;
• Distributed Amplifiers

\[
\gamma_g = \alpha_g + j\beta_g \approx \frac{\omega^2 R_i C^2_{gs} Z_g}{2 l_g} + j\omega \sqrt{L_g \left( C_g + \frac{C_{gs}}{l_g} \right)}
\]

\[
\gamma_d = \alpha_d + j\beta_d \approx \frac{Z_d}{2R_{ds} l_d} + j\omega \sqrt{L_d \left( C_d + \frac{C_{ds}}{l_d} \right)}
\]

\[
I_o = -\frac{g_m V_i}{2} \frac{e^{-N\gamma_{ds} l_g} - e^{-N\gamma_{dl} l_d}}{e^{-\gamma_{ds} l_g} - e^{-\gamma_{dl} l_d}}
\]

\[
G = \frac{P_{out}}{P_{in}} = \frac{g_m^2 Z_d Z_g}{4} \left| \frac{e^{-N\gamma_{ds} l_g} - e^{-N\gamma_{dl} l_d}}{e^{-\gamma_{ds} l_g} - e^{-\gamma_{dl} l_d}} \right|^2
\]

\[
\approx \frac{g_m^2 Z_d Z_g}{4} \frac{(e^{-N\alpha_d l_g} - e^{-N\alpha_g l_d})^2}{(e^{-\alpha_d l_g} - e^{-\alpha_g l_d})^2}
\]

\[
N_{opt} = \frac{\ln\left( \frac{\alpha_g l_g}{\alpha_d l_d} \right)}{\alpha_g l_g - \alpha_d l_d}
\]

Example 11.7

Assume \( Z_d = Z_g = Z_0 = 50 \Omega \),

\( R_i = 10 \Omega \), \( R_{ds} = 300 \Omega \),

\( C_{gs} = 0.27 \) pF, \( g_m = 35 \) mS.

\( N = 2, 4, 8, 16; \ f = 1 \sim 18 \) GHz.
Power Amplifiers

- Characteristics of Power Amplifiers and Amplifier Classes
  - Important considerations for power amplifier: efficiency, gain, intermodulation products, thermal effects.
  - LNA, fixed gain Amp: small-signal amplifiers (transistor: linear device)

Amplifier efficiency:
\[ \eta = \frac{P_{\text{out}}}{P_{\text{DC}}} \]

Power added efficiency:
\[ \eta_{\text{PAE}} = \text{PAE} = \frac{P_{\text{out}} - P_{\text{in}}}{P_{\text{DC}}} \]

\[ = \left(1 - \frac{1}{G}\right)P_{\text{out}} = \left(1 - \frac{1}{G}\right)\eta \]

Compressed Gain:
\[ G_1(\text{dB}) = G_0(\text{dB}) - 1 \]

→ Linearity, intermodulation distortion

Class A: linear, always conduct
  maximum efficiency 50%

Class B: conduct only 1/2 cycle
  push-pull type, 78%

Class C: near cutoff more than 1/2 cycle
  efficiency near 100%, resonant circuit
Power Amplifiers

- **Large-Signal Characterization of Transistors**
  - For power near or larger than $P_1$, $S$ parameters will depend on input power level, output impedance, frequency, bias condition, temperature.

- **Table 11.1** Small-Signal $S$ Parameters and Large-Signal Reflection Coefficients (Silicon Bipolar Power Transistor)

<table>
<thead>
<tr>
<th>$f$ (MHz)</th>
<th>$S_{11}$</th>
<th>$S_{12}$</th>
<th>$S_{21}$</th>
<th>$S_{22}$</th>
<th>$\Gamma_{SP}$</th>
<th>$\Gamma_{LP}$</th>
<th>$G_p$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>$0.76 \angle 176^\circ$</td>
<td>$4.10 \angle 76^\circ$</td>
<td>$0.065 \angle 49^\circ$</td>
<td>$0.35 \angle -163^\circ$</td>
<td>$0.856 \angle -167^\circ$</td>
<td>$0.455 \angle 129^\circ$</td>
<td>13.5</td>
</tr>
<tr>
<td>900</td>
<td>$0.76 \angle 172^\circ$</td>
<td>$3.42 \angle 72^\circ$</td>
<td>$0.073 \angle 52^\circ$</td>
<td>$0.35 \angle -167^\circ$</td>
<td>$0.747 \angle -177^\circ$</td>
<td>$0.478 \angle 161^\circ$</td>
<td>12.0</td>
</tr>
<tr>
<td>1000</td>
<td>$0.76 \angle 169^\circ$</td>
<td>$3.08 \angle 69^\circ$</td>
<td>$0.079 \angle 53^\circ$</td>
<td>$0.36 \angle -169^\circ$</td>
<td>$0.797 \angle -187^\circ$</td>
<td>$0.491 \angle 185^\circ$</td>
<td>10.0</td>
</tr>
</tbody>
</table>
Power Amplifiers

- Load-pull contours
  Automated measurement with electromechanical stub tuners

Nonlinear models: $C_{gs}$, $g_m$, $C_{gd}$, $R_{ds}$

Temperature!!!

Figure 11-16 (p. 572)
Constant output power contours versus load impedance for a typical power FET.
Example 11.8 Design of a Class A Power Amplifier

Design a power amplifier at 900 MHz using Motorola MRF858S NPN silicon BJT.

\[ S_{11} = 0.94 \angle 164^\circ, S_{12} = 0.031 \angle 59^\circ, S_{21} = 1.222 \angle 43^\circ, S_{22} = 0.57 \angle -165^\circ \]

For class A operation at \( V_{CE} = 24V \) and \( I_C = 0.5A, P_{1dB,o} = 3.6W, G_{1dB} = 12dB \),

\( Z_{in} = 1.2 + j3.5\Omega, Z_{out} = 9.0 + j14.5\Omega \), design input and output matching circuits to give 3W output power.

**Solution**

From small - signal S - parameter, \(|\Delta| = 0.546 < 1, K = 1.177 > 1 \rightarrow \text{unconditional stable} \)

From \( Z_{in} \) and \( Z_{out} \rightarrow \Gamma_{in} = 0.953 \angle 172^\circ, \Gamma_{out} = 0.716 \angle -174^\circ \)

From small - signal S - parameter, \( \Gamma_S = 0.953 \angle -172^\circ \approx \Gamma_{in}^*, \Gamma_L = 0.712 \angle 134^\circ \approx \Gamma_{out}^* \)

For \( P_{out} = 3W, P_{in} = P_{out} - G_{1dB} = 22.8dBm = 189mW \),

\[ \eta_{PAE} = \frac{P_{out} - P_{in}}{VI} = \frac{3 - 0.189}{24 \times 0.5} = 23.4\% \]
Power Amplifiers

Figure 11-17 (p. 574)  RF circuit for the amplifier of Example 11.8.