Oscillators and Mixers
Oscillators and Mixers

- RF oscillators
- Microwave oscillators
- Oscillator phase noise
- Frequency multipliers
- Mixers

**Important considerations for oscillators:**
- Tuning range (MHz/V)
- Frequency stability (PPM/°C)
- AM and PM noise (dBc/Hz below carrier, offset from carrier)
- Harmonics (dBc below carrier)
RF Oscillators

\[ V_o(\omega) = AV_i(\omega) + H(\omega)AV_o(\omega) \]

\[ \rightarrow V_o(\omega) = \frac{A}{1 - AH(\omega)} V_i(\omega) \]

Nyquist criterion (Barkhausen criterion)

\[ 1 - AH(\omega) = 0 \] at a particular frequency

\[ \rightarrow \text{zero input but non-zero output} \]
RF Oscillators

• General Analysis
  – Hartley, Colpitts, Clapp, Pierce oscillator circuits

Figure 12-2 (p. 579)  General circuit for a transistor oscillator. The transistor may be either a bipolar junction transistor or a field effect transistor. This circuit can be used for common emitter/source, base/gate, or collector/drain configurations by grounding either $V_2$, $V_1$, or $V_4$, respectively. Feedback is provided by connecting node $V_3$ to $V_4$. 
RF Oscillators

\[
\begin{bmatrix}
(Y_1 + Y_3 + G_i) & -Y_3 & -Y_3 & 0 \\
-(Y_1 + G_i + g_m) & (Y_1 + Y_2 + G_i + G_0 + g_m) & -Y_2 & -G_0 \\
-Y_3 & -Y_2 & (Y_2 + Y_3) & 0 \\
g_m & -(G_0 + g_m) & 0 & G_0
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4
\end{bmatrix} = 0
\]

- Oscillators Using a Common Emitter BJT

\[
\begin{bmatrix}
(Y_1 + Y_3 + G_i) & -Y_3 \\
(g_m - Y_3) & (Y_2 + Y_3)
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V
\end{bmatrix} = 0
\]

where \( V = V_3 = V_4 \)

\[
\begin{vmatrix}
G_i + j(B_1 + B_3) & -jB_3 \\
g_m - jB_3 & j(B_2 + B_3)
\end{vmatrix} = 0
\]

\[
\frac{1}{B_1} + \frac{1}{B_2} + \frac{1}{B_3} = 0
\]

\[
\frac{1}{B_3} + \left(1 + \frac{g_m}{G_i}\right) \frac{1}{B_2} = 0
\]

\[
X_1 = 1/B_1, \ X_2 = 1/B_2, \ X_3 = 1/B_3:
\]

\[
X_1 + X_2 + X_3 = 0
\]

\[
X_1 = \frac{g_m}{G_i} X_2
\]

\( X_1, X_2 \) : the same sign

\( X_3 \) : opposite type component.

Colpitts oscillator: \( X_1, X_2 \) – capacitors

\( X_3 \) – inductor

Hartley oscillator: \( X_1, X_2 \) – inductors

\( X_3 \) – capacitor
RF Oscillators

Colpitts oscillator:

\[ X_1 = -1/\omega_0 C_1, \quad X_2 = -1/\omega_0 C_2, \quad X_3 = \omega_0 L_3 \]

\[ -\frac{1}{\omega_0} \left( \frac{1}{C_1} + \frac{1}{C_2} \right) + \omega_0 L_3 = 0 \]

\[ \omega_0 = \sqrt{\frac{1}{L_3} \left( \frac{C_1 + C_2}{C_1 C_2} \right)} \]

\[ \frac{C_2}{C_1} = \frac{g_m}{G_i} \]

Hartley oscillator:

\[ X_1 = \omega_0 L_1, \quad X_2 = \omega_0 L_2, \quad X_3 = -1/\omega_0 C_3 \]

\[ \omega_0 (L_1 + L_2) - \frac{1}{\omega_0 C_3} = 0 \]

\[ \omega_0 = \sqrt{\frac{1}{C_3 (L_1 + L_2)}} \]

\[ \frac{L_1}{L_2} = \frac{g_m}{G_i} \]
RF Oscillators

- Oscillators Using a Common Gate FET

\[
\begin{bmatrix}
(Y_1 + Y_2 + g_m + G_o) & -(Y_2 + G_o) \\
-(G_o + g_m + Y_2) & (Y_2 + Y_3 + G_o)
\end{bmatrix}
\begin{bmatrix}
V_2 \\
V
\end{bmatrix} = 0
\]

where \( V = V_3 = V_4 \)

\[
\begin{bmatrix}
(g_m + G_o) + j(B_1 + B_2) & -G_o - jB_2 \\
-(g_m + G_o) - jB_2 & G_o + j(B_2 + B_3)
\end{bmatrix} = 0
\]

\[
\begin{cases}
\frac{1}{B_1} + \frac{1}{B_2} + \frac{1}{B_3} = 0 \\
\frac{G_o}{B_3} + \frac{g_m}{B_1} + \frac{G_o}{B_1} = 0 \\
X_1 = \frac{1}{B_1}, \ X_2 = \frac{1}{B_2}, \ X_3 = \frac{1}{B_3}: \\
X_1 + X_2 + X_3 = 0 \\
\frac{X_2}{X_1} = \frac{g_m}{G_o}
\end{cases}
\]

- \( X_1, X_2 \) : the same sign
- \( X_3 \) : opposite type component.

- Colpitts oscillator: \( X_1, X_2 \) – capacitors
  \( X_3 \) – inductor

\[
\omega_0 = \sqrt{\frac{1}{L_3} \left( \frac{C_1 + C_2}{C_1 C_2} \right)}, \quad \frac{C_1}{C_2} = \frac{g_m}{G_0}
\]

- Hartley oscillator: \( X_1, X_2 \) – inductors
  \( X_3 \) – capacitor

\[
\omega_0 = \sqrt{\frac{1}{C_3 (L_1 + L_2)}}, \quad \frac{L_2}{L_1} = \frac{g_m}{G_0}
\]
RF Oscillators

- Practical Considerations
  - Reactance at transistor ports
  - Temperature
  - Bias and decoupling circuitry
  - Inductor loss

Inductor impedance: \( Z_3 = 1/Y_3 = R + j \omega L_3 \)

\[
\omega_0 = \sqrt{\frac{1}{L_3} \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{G_i R}{C_1} \right)} = \sqrt{\frac{1}{L_3} \left( \frac{1}{C_1'} + \frac{1}{C_2} \right)}
\]

\[
C_1' = \frac{C_1}{1 + RG_i}
\]

\[
R = \frac{g_m}{G_i} = \frac{1}{\omega_0^2 C_1 C_2} - \frac{L_3}{L_1}
\]

Example 12.1 Colpitts Oscillator

Design a 50 MHz Colpitts oscillator.

\( \beta = g_m/G_i = 30, \ R_i = 1200 \Omega, \ L_3 = 0.1 \mu H \)

with a \( Q \) of 100. What is the \( Q_{\text{min}} \)?

**Solution**

\[
\frac{C_1 C_2'}{C_1'C_2} = \frac{1}{\omega_0^2 L_3} = 100 \text{ pF} \Rightarrow C_1' = C_2 = 200 \text{ pF}
\]

\[
R = \frac{\omega_0^2 L_3}{Q} = 0.31 \Omega \Rightarrow C_1 = C_1' (1 + RG_i) \approx 200 \text{ pF}
\]

\[
R = \frac{1 + \beta}{\omega_0^2 C_1 C_2} - L_3 \Rightarrow 372 < 7352
\]

\[
R_{\text{max}} = \frac{1}{R_i} \left( \frac{1 + \beta}{\omega_0^2 C_1 C_2} - \frac{L_3}{C_1} \right) = 6.13 \Omega
\]

\[
Q_{\text{min}} = \frac{\omega_0 L_3}{R_{\text{max}}} = 5.1
\]
RF Oscillators

- Crystal Oscillator

![Crystal Oscillator Diagram](image)

\[ \omega_s = \frac{1}{\sqrt{LC}} \]

\[ \omega_p = \frac{1}{\sqrt{L \left( \frac{C_0 C}{C_0 + C} \right)}} \]

\[ = \omega_s \sqrt{1 + \frac{C}{C_0}} \]

**Figure 12-4 (p. 584)** (a) Equivalent circuit of a crystal. (b) Input reactance of a crystal resonator.

**Figure 12-5 (p. 585)** Pierce crystal oscillator circuit.
Microwave Oscillators

\[ Z_{in}(I, j\omega) = R_{in}(I, j\omega) + jX_{in}(I, j\omega) \]
\[ (Z_L + Z_{in})I = 0 \]
\[ R_L + R_{in} = 0 \]
\[ X_L + X_{in} = 0 \]

\[ \Rightarrow \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{Z_{in} + Z_0}{Z_{in} - Z_0} = \frac{1}{\Gamma_{in}} \]

\[ \delta I \quad \text{and} \quad \delta s \quad (s = \alpha + j\omega) \]
\[ Z_T(I, s) = Z_{in}(I, s) + Z_L(s) \]
\[ Z_T(I, s) = Z_T(I_0, s_0) + \frac{\partial Z_T}{\partial \delta} \bigg|_{s_0, I_0} \delta s \]
\[ + \frac{\partial^2 Z_T}{\partial \delta^2} \bigg|_{s_0, I_0} \delta I = 0 \]
\[ Z_T(I_0, s_0) = 0 \]
\[ \delta s = \delta \alpha + j\delta \omega = \frac{-\partial Z_T}{\partial \delta} \bigg|_{s_0, I_0} \delta I \]
\[ = \frac{-j(\partial Z_T / \partial I)(\partial Z_T^* / \partial \omega)}{\left| \partial Z_T / \partial \omega \right|^2} \delta I \]
\[ \Rightarrow \text{Im} \left\{ \frac{\partial Z_T}{\partial I} \frac{\partial Z_T^*}{\partial \omega} \right\} < 0 \]
Microwave Oscillators

\[
\frac{\partial R_T}{\partial I} \frac{\partial X_T}{\partial \omega} - \frac{\partial X_T}{\partial I} \frac{\partial R_T}{\partial \omega} > 0
\]

For a passive load, \( \frac{\partial R_L}{\partial I} = \frac{\partial X_L}{\partial I} = \frac{\partial R_L}{\partial \omega} = 0 \)

\[\Rightarrow \frac{\partial R_{in}}{\partial I} \frac{\partial}{\partial \omega} (X_L + X_{in}) - \frac{\partial X_{in}}{\partial I} \frac{\partial R_{in}}{\partial \omega} > 0\]

\[\frac{\partial R_{in}}{\partial I} > 0 \Rightarrow \frac{\partial (X_L + X_{in})}{\partial \omega} >> 0\]

\[\Rightarrow \text{high-Q result in max oscillator stability}\]

**Example 12.2 Negative-Resistance Oscillator Design**

One-port oscillator having \( \Gamma_{in} = 1.25 \angle 40^\circ \), \( Z_0 = 50 \Omega \). \( f = 6 \text{ GHz} \). Design a load matching network for a \( 50 \Omega \) load impedance.

**Solution**

\[Z_{in} = -44 + j123 \Omega\]

\[Z_L = 44 + j123 \Omega\]

*Figure 12-7 (p. 587)*  Load matching circuit for the one-port oscillator of Example 12.2.
Microwave Oscillators

- Transistor Oscillators

![Diagram of microwave oscillator circuit]

Choose $R_L + R_{in} < 0$

In practice, $R_L = -R_{in}/3$ is typically used.

$X_L = -X_{in}$

$\Gamma_L \Gamma_{in} = 1 \implies \frac{1}{\Gamma_L} = \Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_T}{1 - S_{22} \Gamma_T} = \frac{S_{11} - \Delta \Gamma_T}{1 - S_{22} \Gamma_T}$

$\Gamma_T = \frac{1 - S_{11} \Gamma_L}{S_{22} - \Delta \Gamma_L}$

$\Gamma_{out} = \frac{S_{22} - \Delta \Gamma_L}{1 - S_{11} \Gamma_L}$

$\implies \Gamma_T \Gamma_{out} = 1$, $Z_T = -Z_{out}$
Microwave Oscillators

- Example 12.3 Transistor Oscillator Design

Design a transistor oscillator at 4 GHz using GaAs FET (CG) with a 5 nH inductor in series with the gate to increase the instability. Matching to 50 $\Omega$ ($S$ parameters: $S_{11} = 0.72\angle -116^\circ$, $S_{21} = 2.6\angle 76^\circ$, $S_{12} = 0.03\angle 57^\circ$, $S_{22} = 0.73\angle -54^\circ$).

Solution

$S'_{11} = 2.18\angle -35^\circ$, $S'_{21} = 2.75\angle 96^\circ$, $S'_{12} = 1.26\angle 18^\circ$, $S'_{22} = 0.52\angle 155^\circ$.


c_T = \frac{(S'_{22} - \Delta' S'^{*}_{11})^*}{|S'_{22}|^2 - |\Delta'|^2} = 1.08\angle 33^\circ,

R_T = \frac{|S'_{12} S'_{21}|}{|S'_{22}|^2 - |\Delta'|^2} = 0.665

Select $\Gamma_T = 0.59\angle -104^\circ$

$Z_T = 20 - j35 \Omega$

- Figure 12-9a (p. 589) Circuit design for the transistor oscillator of Example 12.3. (a) Oscillator circuit.
Microwave Oscillators

$$\Gamma_{in} = S'_{11} + \frac{S'_{12} S'_{21} \Gamma_T}{1 - S'_{22} \Gamma_T} = 3.96 \angle -2.4^\circ$$

or $$Z_{in} = -84 - j1.9 \, \Omega.$$  

$$Z_L = \frac{-R_{in}}{3} - j \, X_{in} = 28 + j1.9 \, \Omega$$  

$$\Rightarrow 90 \, \Omega \text{ with } 0.262\lambda \text{ length line}$$

**Figure 12-9b (p. 589)**  
(b) Smith chart for determining $$\Gamma_T.$$
Microwave Oscillators

- Dielectric Resonator Oscillators

Figure 12-10 (p. 590)
(a) Geometry of a dielectric resonator coupled to a microstripline; (b) equivalent circuit.
Figure 12-11 (p. 591)
(a) Dielectric resonator oscillator using parallel feedback; (b) dielectric resonator oscillator using series feedback.
Oscillator Phase Noise

**Figure 12-13 (p. 594)** Output spectrum of a typical RF oscillator.

**Figure 12-18 (p. 598)** Illustrating how local oscillator phase noise can lead to the reception of undesired signals adjacent to the desired signal.

\[
L(f_m) \text{ (dBc/Hz)} = C(\text{dBm}) - S(\text{dB}) - I(\text{dBm}) - 10\log(B),
\]

**Figure 12-15 (p. 596)** Noise power versus frequency for an amplifier with an applied input signal.
Mixers

Figure 12-30 (p. 617)
Frequency conversion using a mixer. (a) Up-conversion. (b) Down-conversion.
Mixers

Figure 12-31 (p. 621)
(a) Circuit for a single-ended diode mixer.
(b) Idealized equivalent circuit.
MIXERS

Figure 12-32 (p. 622)
Variation of FET transconductance versus gate-to-source voltage.

Figure 12-33 (p. 623)
Circuit for a single-ended FET mixer.

Figure 12-34 (p. 623)
Equivalent circuit for the FET mixer of Figure 12.33.
Mixers

Figure 12-35 (p. 625)
Balanced mixer circuits. (a) Using a 90° hybrid. (b) Using a 180° hybrid.
Mixers
Mixers

Figure 12-37 (p. 628) Circuit for an image reject mixer.
Mixers

Figure 12-38 (p. 629)  Double balanced mixer circuit.
Mixers

Figure 12-39 (p. 630)
A differential FET mixer.