Understanding brand performance measures: using Dirichlet benchmarks

Andrew S.C. Ehrenberg, Mark D. Uncles, Gerald J. Goodhardt

Abstract

Sales of a brand are determined by measures such as how many customers buy the brand, how often, and how much they also buy other brands. Scanner panel operators routinely report these “brand performance measures” (BPMs) to their clients. In this position paper, we consider how to understand, interpret, and use these measures. The measures are shown to follow well-established patterns. One is that big and small brands differ greatly in how many buyers they have, but usually far less in how loyal these buyers are. The Dirichlet model predicts these patterns. It also provides a broader framework for thinking about all competitive repeat-purchase markets—from soup to gasoline, prescription drugs to aviation fuel, where there are large and small brands, and light and heavy buyers, in contexts as diverse as the United States, United Kingdom, Japan, Germany, and Australasia.

Numerous practical uses of the framework are illustrated: auditing the performance of established brands, predicting and evaluating the performance of new brands, checking the nature of unfamiliar markets, of partitioned markets, and of dynamic market situations more generally (where the Dirichlet provides theoretical benchmarks for price promotions, advertising, etc.). In addition, many implications for our understanding of consumers, brands, and the marketing mix logically follow from the Dirichlet framework. In repeat-purchase markets, there is often a lack of segmentation between brands and the typical consumer exhibits polygamous buying behavior (though there might be strong segmentation at the category level). An understanding of these applications and implications leads to consumer insights, imposes constraints on marketing action, and provides norms for evaluating brands and for assessing marketing initiatives.

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1. Introduction

1.1. Overview

Sales of Folgers, P&G’s leading brand of instant coffee in the United States, depended on it being bought three times a year on average by 11% of households. Roughly 12% of these customers were 100% loyal. Therefore, most customers bought other brands as well. In total, they did so about as often as they bought Folgers itself—a “share of category requirements” (SCRs) of only 50%.

Such performance measures are routinely tabulated in marketing research reports and cited in marketing plans. But to interpret the figures, benchmarks and norms are needed. For example, is it that as many as 12% or only 12% are 100% loyal? The thesis of this paper is that there is a simple and general answer to such a question: most performance measures of most brands are just about normal. And in general, sales of a brand are largely determined by much the same predictable patterns of buyer behavior.

Consumers are seen as mostly choosing from personal split-loyalty repertoires, typically buying one brand more often than another. Within such a framework of steady but divided loyalties, specific purchases then occur in a seemingly irregular or even “as-if-random” manner. These personal repertoires of brands differ from one consumer or household to the next. Yet such heterogeneous behavior aggregates to brand performance measures (BPMs) that follow much the same “lawlike” pattern from brand to brand. Key examples are that any loyalty-related measure is usually much alike for different brands, that a brand typically has many light buyers, and that few of its customers are 100% loyal over a sequence of purchases. Superimposed are regular relationships with market share, e.g., for purchase rates and for brand switching.
These “lawlike” patterns have been found in over 50 varied product and service categories from soap to soup, motor cars, prescription drugs, media usage, etc., and in different countries and at different points in time, as is discussed in Section 2. These patterns are in turn closely predictable from a single and parsimonious model, the Dirichlet (or “NBD-Dirichlet in full”). BPMs are all determined in the model just by the brand’s market share and only indirectly by the marketing mix or consumer-related factors acting through the market share. This is outlined in Section 3, with the theory being summarized in Appendix A.

The Dirichlet model postulates that each consumer has a certain propensity—a probability in the model—to buy a given brand. This probability is assumed to be steady for the time being but differing across heterogeneous consumers. The model is defined for steady state and unpartitioned markets where market shares are stationary and there is no clustering of particular brands. However, this does not imply that all markets should be stationary and nonpartitioned (although many often are). The model purports only to describe what markets are like when they are near steady and nonpartitioned. But, even where the market is not quite steady, or where there is some clustering, the Dirichlet mostly still holds and it provides useful benchmarks.

The Dirichlet-type regularities and associated model still cause surprise. Can there be “lawlike” patterns or even just a “near-steady state” in markets that are constantly subjected to competitive inputs, technological innovations, and environmental changes? Moreover, even if near-steady state markets do exist, can they be of any interest to marketing practitioners who are always trying to change that steady state? The answers to both these questions appear to be a clear “yes.” Given that steady state Dirichlet norms or benchmarks occur widely, it is possible to address many practical marketing issues (Section 4):

(i) Auditing a brand’s performance to see whether or not it is in fact normal.
(ii) Identifying market partitioning and other departures from the basic norms.
(iii) Assessing and interpreting dynamic non-steady-state situations such as price promotions, new brands, and sales trends.

The Dirichlet framework—the empirical patterns, the mathematical model, and the underlying theory—also has wider implications for our understanding of consumers, brands, differentiation and positioning, advertising, sales promotions, etc. (Section 5).

The empirical Dirichlet-type regularities have seldom been criticized. But the patterns are not always well-known or understood, and their theoretical interpretation and practical use is at times controversial. Therefore, our aim in this position paper is to bring together, update, and synthesize these findings for marketing researchers, practitioners, teachers/students, and potential critics.

1.2. A new-brand case

Many marketing analysts seem to be puzzled how a steady state model like the Dirichlet, which contains no explicit “decision variables,” might nonetheless be of use in dynamic situations such as launching a new brand. To illustrate, we outline a simple new brand case based on an amalgam of practical experience. This shows the implications of knowing (or not knowing) the common Dirichlet-type market pattern that brand loyalty varies little, or relatively little, between competitive brands.

1.2.1. The new Brand X

In considering a new instant coffee brand, “X,” a U.S. corporation had decided on a market share of 5% or eight purchases a year per 100 households. This target had been set by the management after having considered various marketing-mix expenditures and estimated revenues. The management was then given the choice of two marketing policies, A and B.

Policy A was to position Brand X as a niche brand since 16% of consumers had rated it very high in placement tests. X was therefore to be targeted at a small heavy-buying segment who really liked the brand, with some 1% of the population buying it about eight times a year (thus, satisfying the sales target of eight purchases per 100 households). As a niche brand, X was to be premium priced, with loyalty-building promotions and advertising in up-market media. Distribution was also to be up-market, without expensive trade deals, relying on product and advertising pull rather than trade push.

Policy B was an extreme mass marketing positioning. X would be an add-on or variety brand based on its special product formulation. It would reach the preset sales target of eight by being bought by some 8% of the population but only about once a year. It would be competitively priced, with awareness-building promotions, “try-something-different” advertising in mass media, and trade support bought by slotting allowances and eye-catching merchandising.

The management then asked for a check against consumer panel data. The results in Table 1 showed that both policies A and B were totally out of step with the market. Both the observed and theoretical purchase rates for the different brands were very similar at about 3. This meant that the new Brand X also should be expected to stabilize at about 3.

Interpolating more precisely, the analysts concluded that if the new Brand X did reach its ultimate targeted share of 5% (or eight purchases per 100 households), it would be bought about 2.7 times a year by its buyers and hence by 3% of all consumers (since 8/2.7 = 3). Therefore, a realistic sales equation (where sales = percent buying × rate of buying) would be 3% buying 2.7 times (a normal brand) compared with the hypothesized strategies of policy A where 1% buy eight times (a niche brand) or policy B where 8% buy once (an add-on brand).
The NPD team now queried whether the similar average purchase frequencies of about three in Table 1 were not an unexpected coincidence, with at least one big exception (for Maxim). The analysts countered that the close fit of the well-established theoretical norms ($T$) showed that the pattern was neither accidental nor unexpected. They added that to break out of this pattern, $X$ would have to differ far more from the existing brands than they differed from each other. Yet the marketing plan for $X$ had claimed nothing of the kind. From this, management concluded that the new brand would not reach its 5% sales target via either the niche policy A or the mass market policy B. They also felt that $X$’s allocated budget was not sufficient for an achievable me-too target. As a result, plans for Brand $X$ were dropped. This forestalled that it would be one of the traditional “four out of five new brands that fail.”

1.2.2. A postmortem

Later, senior management asked their new marketing-insights director to critique the earlier planning. First, she queried that two totally different policies, niche versus variety branding, had been put forward as alternatives for the same functional Brand $X$. This struck her as an inappropriate “anything goes” approach to planning. Second, she questioned whether a me-too launch should have been ruled out as unattractive compared with launching a brand with a difference (i.e., one with a distinctive built-in selling proposition). She argued that in fact competitive brands are mostly much alike; yet some are able to achieve greatness.

We now step back from this introductory case history to review the main patterns of buyer behavior that have been observed for frequently bought products.

2. Patterns of buyer behavior

The basic finding about individual consumers’ varied buying behavior is that whichever way the data are aggregated, they show regular patterns. These patterns slowly came to be recognized over the years, followed by many replications across different products, years, and countries to develop their generalizability.

2.1. Brand performance measures

The variables analyzed are the standard BPMs used by large consumer panel operators (e.g., ACNielsen, IRI, TNSofres, GfK, etc.) and their clients (e.g., Unilever, P&G, Colgate, Kraft, Nestle, etc.) (Bucklin and Gupta, 1999; Sudman and Wansink, 2002). They are of three kinds, as illustrated in Table 2 for the leading U.S. instant coffee brand.

2.1.1. Brand-size-related measures

The two main measures of size, market share and market penetration, are very different conceptually:

\[ \text{Market share} = \frac{\text{Total purchases of the brand}}{\text{Total purchases of the category}} \]

\[ \text{Penetration} = \frac{\text{The number buying the brand at least once}}{\text{The total number of potential customers}} \]

Folgers market share (24% in terms of purchase occasions) was much the same in any length time period in a typically near-steady market. But the number of customers buying Folgers increased greatly, from about 1% in a week to 11% in a year.

Table 1
Annual penetrations and purchase rates (leading brands of instant coffee)

<table>
<thead>
<tr>
<th>Brand</th>
<th>Market share (%)</th>
<th>Percent buying (per buyer)</th>
<th>Purchases (per buyer)</th>
<th>Percent buying Category</th>
<th>SCR $^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Folgers</td>
<td>24</td>
<td>11</td>
<td>12</td>
<td>3.2</td>
<td>31</td>
</tr>
<tr>
<td>Maxwell House</td>
<td>22</td>
<td>10</td>
<td>11</td>
<td>3.1</td>
<td>24</td>
</tr>
<tr>
<td>Tasters Choice</td>
<td>17</td>
<td>9</td>
<td>9</td>
<td>2.8</td>
<td>21</td>
</tr>
<tr>
<td>Nescafé</td>
<td>11</td>
<td>6</td>
<td>6</td>
<td>2.7</td>
<td>12</td>
</tr>
<tr>
<td>Sanka</td>
<td>9</td>
<td>5</td>
<td>5</td>
<td>3.0</td>
<td>1</td>
</tr>
<tr>
<td>High Point</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2.6</td>
<td>1</td>
</tr>
<tr>
<td>Maxim</td>
<td>1</td>
<td>0.3</td>
<td>0.8</td>
<td>4.5</td>
<td>1</td>
</tr>
<tr>
<td>Brim</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
<td>2.1</td>
<td>1</td>
</tr>
<tr>
<td>Other brands</td>
<td>16</td>
<td>8</td>
<td>8</td>
<td>3.0</td>
<td>21</td>
</tr>
<tr>
<td>The new Brand X</td>
<td>5</td>
<td>–</td>
<td>3</td>
<td>2.7</td>
<td>13</td>
</tr>
</tbody>
</table>

Data: Hallberg (1996)/MRCA; $O =$ observed; $T =$ theoretical Dirichlet predictions.

$^a$ Outlier.

Table 2
Common Brand Performance Measures (BPMs): Folgers

<table>
<thead>
<tr>
<th>USA 1992</th>
<th>Brand size</th>
<th>Loyalty-related measures (annual)</th>
<th>Switching (annual)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Folgers</td>
<td>Market share (%)</td>
<td>Percent buying in a: &amp; Purchases</td>
<td>Percentage of Folgers buyers who also bought $^b$</td>
</tr>
<tr>
<td></td>
<td>Week</td>
<td>Year</td>
<td>SCR</td>
</tr>
<tr>
<td>Observed</td>
<td>24</td>
<td>11</td>
<td>3.2</td>
</tr>
</tbody>
</table>

$^a$ For brand names see Table 1.

$^b$ Share of category requirements.
### 2.1.2. Loyalty-related measures

Folgers customers averaged 3.2 purchases in the year. But half bought the brand only once. These customers bought the category (i.e., any instant) 6.4 times on average, giving the brand an SCR of 50% (i.e., 3.2/6.4).

### 2.1.3. Switching-related measures

Table 2 also shows which other instant coffee brands Folgers’ customers bought once or more in the year; e.g., 31% also bought Maxwell House, and only 1% also bought Maxim.

The question now is what these figures mean. Why are they what they are? Is an 11% annual penetration high or low? Is a SCR of only 50% as threatening as it seems? Does the relatively high purchase duplication of Folgers with Maxwell House mean that these two brands are highly competitive—much more so than with Maxim?

### 2.2. Compared with what: regularities from brand to brand

To interpret these numbers for Folgers, we can compare them with those for other brands. To illustrate, Table 3 compares a dozen performance measures across the top eight brands in 12 varied product categories internationally.

Several patterns are apparent in Table 3 and for each of the 12 categories:

- **Double jeopardy**—Market shares and penetrations decrease greatly, by almost 10-fold, from Brand A to Brand H. In contrast, loyalty- or switching-related measures either stay broadly the same from brand to brand or decrease far less (by factors of 2 or 3 at most). Smaller brands therefore not only have far fewer buyers than the bigger brands but also show somewhat lower average purchase frequencies (i.e., lower repeat buying). This tendency to be “punished twice” just for being small was called “double jeopardy” (DJ) by McPhee (1963), who explained it as statistical selection effect. The DJ effect is however small unless penetrations are very high.

- **Heterogeneity**—Consumers are very heterogeneous. On average, one in six of a brand’s annual customers bought it five or more times (with DJ again: smaller brands have even fewer heavier buyers). But some 50% bought it just once in the year (as for Folgers in Table 2).

- **The natural monopoly effect**—How often customers of a brand bought the whole category increases slightly from 10 to 13 with decreasing market share. This atypical (upward) trend was called “natural monopoly” by McPhee in 1963. It implies that large brands slightly “monopolize” light category buyers.

- **SCRs**—The average SCR of a brand over the year is quite low; e.g., about 27% annually (on average 3.2/12). Any brand’s customers mostly buy other brands, showing multibrand buying behavior.

- **100% loyals**—Only some 12% of a brand’s customers were 100% loyal in the year, with even fewer for a small brand (DJ again). 100% loyals are in fact light users of the whole category (on average about 4 such purchases versus 12 by all of the brand’s customers). They do not have many opportunities to be disloyal.

- **Which other brands are also bought**—Much the same proportions of any particular brand’s customers also bought Brand A (all about 54%). Similarly for C (about 33%) or E (about 19%). The markets in question were therefore nonpartitioned, i.e., with no apparent clustering of some subsets of brands. These switching levels are themselves proportional to the brand’s penetrations—the so-called “duplication of purchase law,” as described in Appendix A.

### Table 3

<table>
<thead>
<tr>
<th>Brands (by share)</th>
<th>Loyalty-related measures (annual)</th>
<th>Switching (annual)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Market Share</td>
<td>Percent buying</td>
</tr>
<tr>
<td></td>
<td>(% Weekly)</td>
<td>Yearly</td>
</tr>
<tr>
<td>A</td>
<td>28</td>
<td>3.6</td>
</tr>
<tr>
<td>B</td>
<td>19</td>
<td>2.5</td>
</tr>
<tr>
<td>C</td>
<td>12</td>
<td>1.5</td>
</tr>
<tr>
<td>D</td>
<td>9</td>
<td>1.2</td>
</tr>
<tr>
<td>E</td>
<td>7</td>
<td>0.9</td>
</tr>
<tr>
<td>F</td>
<td>5</td>
<td>0.7</td>
</tr>
<tr>
<td>G</td>
<td>4</td>
<td>0.6</td>
</tr>
<tr>
<td>H</td>
<td>3</td>
<td>0.4</td>
</tr>
<tr>
<td>Average brand</td>
<td>11</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Categories: Catsup, cereals, cheese, orange juice, household cleaners, laundry detergents (2×), paper towels (2×), take-home beer, toothpaste (2×).

Sources: AGB (UK), Nielsen and IRI (USA), GfK (Germany), TCI (Japan).

a Selection of brands.
b Per brand buyer.
2.3. Conditions under which the patterns occur

The generalizability of each of the above patterns is now widely established across a great variety of very different product and services categories, as shown in Table 4. Anyone with access to consumer panel or similar data can check on such patterns.

3. The Dirichlet model

Given their wide generalizability, it is perhaps unsurprising that these patterns of buyer behavior can all be described by a single statistical model—the NBD-Dirichlet (“Dirichlet” for short)—and that this model is simple, consisting of just a few well-based assumptions. The model was developed by Chatfield and Goodhardt (1975) to account for the known empirical patterns, and subsequently by Bass et al. (1976) on a theoretical “utility” basis. To us, the model’s critical feature is that it is able to describe the various observed brand performance patterns; and in that sense, it also helps explain and predict them.

Table 5 illustrates the Dirichlet model’s close theoretical predictions of the observed data, which were shown in Table 3. The fit for each separate measure can be summarized in two respects: (a) the virtual lack of systematic bias, and (b) the O and T values for the individual brands also differing little (e.g., with an average deviation, or mean absolute deviation (MAD), of less than 2 percentage points for the penetration-type percentages).

3.1. The Dirichlet assumptions

Two broad propositions underlie the model. One concerns consumers. The other concerns brands.

Consumers are thought of as highly experienced. They are therefore no longer easily influenced by a further purchase or seeing yet another advert (i.e., no additional “learning” occurs). Hence, consumers can behave as if they have, for the time being, steady habitual personal purchase propensities—stochastic probabilities in the model—for when they buy and what brands they choose on different purchase occasions. Crucially for the Dirichlet model, a consumer’s probability of buying a particular brand would not be affected by what other brands the consumer is also buying.

Brands are characterized in the model by their purchase probabilities and hence their market shares. Beyond that, the model does not specify whether the brands are functionally differentiated or not, or differently marketed—this is all subsumed in the model’s steady state purchase probabilities. These notions are captured in the theoretical model by five specific distributional assumptions: two assumptions are about consumer heterogeneity for purchase frequency and for brand choice, two about the probabilistic incidence of specific purchases of the product and of a brand, and the final one is about the statistical independence of these two aspects. The individual assumptions are spelt out in Appendix A, along with procedures for calculating the theoretical BPMs. There is strong theoretical as well as empirical support for each of these assumptions.

To use the model, it has to be calibrated for the chosen product category and brand. Only four numerical inputs are needed, typically the penetrations and the average purchase frequencies of the category and of one specific brand. To estimate the BPMs of any brand of particular interest its own market share has also to be inputted (see Table A1 of Appendix A). The model is therefore very parsimonious.
Table 5
Annual observed and theoretical performance measures

<table>
<thead>
<tr>
<th>Brands (by share)</th>
<th>Market share (%)</th>
<th>Loyalty-related measures (annual)</th>
<th>100% loyal</th>
<th>Switching (annual)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Percent buying</td>
<td>Percent buying</td>
<td>Percent buying</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(per buyer)</td>
<td>5+ times</td>
<td>per buyer</td>
</tr>
<tr>
<td>A</td>
<td>28</td>
<td>46</td>
<td>46</td>
<td>3.9</td>
</tr>
<tr>
<td>B</td>
<td>19</td>
<td>35</td>
<td>36</td>
<td>3.6</td>
</tr>
<tr>
<td>C</td>
<td>12</td>
<td>25</td>
<td>25</td>
<td>3.1</td>
</tr>
<tr>
<td>D</td>
<td>9</td>
<td>22</td>
<td>21</td>
<td>2.8</td>
</tr>
<tr>
<td>E</td>
<td>7</td>
<td>14</td>
<td>16</td>
<td>3.2</td>
</tr>
<tr>
<td>F</td>
<td>5</td>
<td>12</td>
<td>11</td>
<td>2.7</td>
</tr>
<tr>
<td>G</td>
<td>4</td>
<td>11</td>
<td>10</td>
<td>2.7</td>
</tr>
<tr>
<td>H</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td>3.2</td>
</tr>
<tr>
<td>Average brand</td>
<td>11</td>
<td>21</td>
<td>21</td>
<td>3.2</td>
</tr>
</tbody>
</table>

O = Observed as in Table 3; T = theoretical Dirichlet predictions.

* Consistently low—see discussion of systematic discrepancies in Section 3.2.

3.2. Deviations from the model

Other variables such as marketing-mix inputs or consumer attributes do not have to be explicitly specified. This is because the model is for the steady state and assumes that these effects will usually already have been subsumed in the brands’ market shares, which in turn affect the brand’s other performance measures. Alternatively, they will show up as discrepancies in the model predictions, which then need to be explained (e.g., Bhattacharya et al., 1996; Bhattacharya, 1997; Ehrenberg, 1988; Kahn et al., 1988).

Given the large consumer panels that are used nowadays (e.g., 10,000+ households), many of the reported discrepancies from the model will be “significant” (i.e., not just a sampling error). Moreover, a 5% difference from an average discrepancy from the model will be “significant” (i.e., not just a sampling error). Nevertheless, some systematic discrepancies have been reported, for example:

- Quarter-by-quarter repeat buying can be overpredicted (by 5 to 10 percentage points—Ehrenberg, 1988). In addition, repeat rates over nonadjacent periods tend to erode somewhat by eight percentage points over a year (East and Hammond, 1996). This is a real departure from a “steady state” and can be regarded as a “slightly leaky bucket.” Follow-up work is needed.

- For some market leaders, annual purchase frequencies are a unit or so higher than predicted. But this occurs only occasionally (Ehrenberg et al., 1990; Fader and Schmittlein, 1993; Reibstein and Farris, 1995). It may be due to large brands being more likely to be kept in stock and may not be as important as is sometimes thought.

- An early systematic discrepancy was the so-called “variance discrepancy.” This was found to reflect a shortfall of very frequent buyers, which was “explained away” as an artifact because few people buy a typical grocery product more than once a week or more than 13 times in a quarter (Chatfield, 1967; Ehrenberg, 1959).

- The annual purchase rates of 100% loyal buyers are consistently underpredicted (e.g., by a purchase or two for each of the 100 brands in Table 5). This is so far unexplained. But the discrepancy usually varies relatively little from brand to brand (except with market share) and has not been of diagnostic (i.e., “differentiating”) marketing value so far.

- The distribution of light, medium, and heavy buyers of a total category (or a subcategory) is sometimes a little “flatter” than predicted by the NBD part of the Dirichlet model (i.e., there are slightly too many “medium” buyers in the model). The reasons are not yet understood. The more limited “empirical Dirichlet” model can then be fitted instead, although at the cost of not being able to predict theoretical length-of-time period effects, such as penetration growth (Ehrenberg, 1988).

Such known discrepancies seldom curtail the model’s practical use. Attempts to improve or elaborate the Dirichlet model have so far not resulted in major gains in either predictive power or parsimony (see Appendix A).
4. Practical applications

We now review various practical uses to which the Dirichlet framework has so far been put. Examples are auditing the performance of established brands, predicting and evaluating that of new brands, and checking the nature of unfamiliar markets, of partitioned markets, and of dynamic market situations more generally. In essence, the Dirichlet provides theoretical benchmarks for assessing price promotions or advertising, say, one factor at a time, instead of using all-in-one marketing mix “decision-oriented” modeling (e.g., contrast Leeflang and Wittink, 2000 with Ehrenberg et al., 2000).

4.1. Brand performance audits

A common use of the Dirichlet is to assess how existing brands are performing. Thus, it was not clear in Table 2 whether any of the figures observed for Folgers were high or low (or “good” or “bad”). In practice, the Dirichlet norms showed that all but one of the measures in the table was normal, as shown in Table 5. In another study, 34 product categories were systematically assessed to see if BPMs for P&G brands were as expected (Uncles et al., 1994, 1995).

Isolated deviations occur but often tend to have fairly simple explanations, as in the following five cases:

• In Table 1 for U.S. instant coffee, Maxim’s very high annual purchase rate of 4.5 was found to be due to two “outliers” (two households making over 30 purchases each, as against the average of about 3—perhaps the households were running a bridge club!).

• Brim also had a somewhat low average purchase frequency of 2.1 (Table 1). This had not occurred the year before (the first thing to check). It was therefore not a general characteristic of the brand but a one-off effect due perhaps to a fire in a warehouse (“bad”) or an exceptional promotion with an extralarge bonus of one-off buyers (“good” presumably). An analysis excluding the promoted month would have shown this more explicitly (but the raw data were no longer available).

• For instant coffee in the UK, the market leader Nescafé has over the years had a somewhat high buying rate, even given Nescafé’s high 34% share (possibly this results from the occasional brand leader effect already mentioned).

• A remarkably high rate of new medical prescriptions occurred in 1986 for Squibb’s cardiovascular drug Capoten: 10 new prescriptions a year per prescribing doctor instead of the norm of about 5. This was traced to doctors who were provided with a “free” PC if they prescribed Capoten often enough for medical monitoring purposes (this might seem a highly successful loyalty inducement but would soon become financially crippling). The prescribing rate fell back to normal when the inducement was withdrawn (Stern and Ehrenberg, 1995).

• For private labels, the Dirichlet consistently shows somewhat higher buying frequencies. This was probably due to their inherent nonavailability in other chains, as was tested in two ways: by grouping PLs together as a megabrand and by analyzing a PL within its own store chain. In the outcome, private labels seemed to show normal repeat-buying loyalty (see Uncles and Ellis, 1989; Bound and Ehrenberg, 1997), but more work is needed.

4.2. Extensions to new conditions

The Dirichlet also provides benchmarks when analyzing data for another year, country, or category. Instead of unfocused data mining, it is simpler to check whether the Dirichlet patterns recur. This took place in all the analyses listed in Table 4, some for very different kinds of markets or market conditions. Examples are:

• Geographical extensions—to Australasia and Japan (Wright et al., 1998; Kau et al., 1998) and to markets in “developing” countries (Bennett, 2000).

• Unusually frequently bought categories—gasoline, for example, which is bought on average as often as once a week and has distinctive solus-site retail distribution (Scriven and Ehrenberg, 1994).

• Doctors’ medical prescriptions—where the doctor neither buys, pays for, nor consumes the product (Stern, 1995; Stern and Ehrenberg, 1995).

• Durables and services—repurchase of PCs (Long and Ehrenberg, 1998) and cars (Ehrenberg and Bound, 2000; Colombo et al., 2000) and frequent flying by business travelers (Harris, 2003).

• “Impulse” purchases—made for immediate personal consumption (McDonald and Ehrenberg, 2002).

• Television—repeat viewing and switching for programs and channels (Barwise and Ehrenberg, 1998; Goodhardt et al., 1987).

• Industrial purchasing and procurement—isolated cases so far, such as chemical additives, paper and packaging (Easton, 1980), aviation fuel contracts (Uncles and Ehrenberg, 1990b), and ready mix concrete (Pickford and Goodhardt, 2000).

• Store choice—repeat visits and switching between grocery outlets (Kau and Ehrenberg, 1984; Kau et al., 1998; Uncles and Ehrenberg, 1990a; Uncles and Hammond, 1995; Wrigley and Dunn, 1984, 1985), womenswear retailing (Brewis-Levie and Harris, 1999), and recently on-line electronic shopping (Danaher et al., in press; Moe and Fader, 2001).

• SKU—customer retention and switching between stockkeeping units, such as specific combinations of pack sizes, flavors, and model specification (as for cars or PCs). Whereas brands are often similar (“the commodity with a name”), SKUs unquestionably differ functionally (e.g., large versus small pack sizes). Early work shows
that the Dirichlet-type patterns nonetheless still predominate (Singh et al., 2000).

4.3. Market partitioning

Markets for directly substitutable brands are usually not partitioned; that is, they show no special clustering of particular brands. But some subcategories exist; for example, coffee by ground and instant, decaffeinated and regular. Each of these functional attributes may then attract a special “segmented” following, with consumer choice showing marked clustering for the more similar items (partitioning). Such clustering is brought out by relating the observed duplications of purchase between pairs of brands to these brands’ penetrations, with the nonpartitioned Dirichlet-related “duplication of purchase law” as a well-established norm. This allows for the penetration of each brand. The “law” usually continues to apply directly not only within each separate partition but also between the partitions, but with differing “duplication coefficients” (i.e., the ratios of the duplication levels and the penetrations—see Table A4 of Appendix A for the calculations).

A striking case of partitioning has been reported for the automobile market, with “luxury cars” not unexpectedly showing up as the most distinct of several clusters (Ehrenberg and Bound, 2000). Table 6 illustrates another instance of complex-yet-simple partitioning, this time for the UK gasoline market between unleaded and leaded variants (Scriven and Ehrenberg, 1994) (for simplicity, just four brands are reported in the table).

The purchase duplications between pairs of brands were relatively high, low, or middling but always decreased with decreasing penetrations:

- **High** between the unled brands in the northwest quadrant (and also the led brands in the southeast quadrant) and decreasing from some 30% to 10% in line with the brands’ penetrations.
- **Low** at about 5% between two different brands of either leaded or unled in the southwest and northeast quadrants, again decreasing with the penetrations. The figures are not zero mainly because of households with two cars, one older and still requiring leaded gasoline.
- **Middling** at about 15% for the same brand unled and leaded (in the southwest and northeast diagonals), again because of 2+ car families buying both variants mainly at the same local gas station.

Here is a case where having prior knowledge of the duplication patterns can be expected to work much better than a statistical multivariate “discovery” technique. Similar points emerged from an exhaustive study of car repurchasing in Europe, where a considerable number of techniques were applied and compared and where a focus on expected duplication patterns greatly assisted the analysis (Colombo et al., 2000).

4.4. New brands

Using the Dirichlet for initial new-brand planning has already been rehearsed in Section 1 (the case of Brand X) (see also Ehrenberg, 1991). Another application is in evaluating a new brand some time after its launch. Campbell’s newish premium priced “Tastes of the World” (TotW) soup in Britain for example showed low loyalty-related measures, compared with its bigger competitors. TotW seemed doomed until their marketing advisors made Campbells aware of the DJ phenomenon, i.e., that repeat-buying levels are predictably lower for smaller brands. This showed that TotW’s low loyalty was in fact normal for its size. TotW’s problem was that it had too few customers.

The general view for new brands is that loyalty grows slowly (e.g., as implied in analyses by “depth of repeat”). But no generalizable results of this have been reported (e.g., Hardie et al., 1998). In contrast, Wright and Sharp (1999), and Ehrenberg and Goodhardt (1968a) much earlier, have reported two isolated cases of near-instant loyalty occurring unexpectedly for two small new brands in Australia and the UK. A more extreme exception was Unilever’s new toilet soap Shield some years ago. This achieved a remarkable 20% share in a very traditional UK market almost immediately at launch, with repeat buying and switching also very high immediately, and indeed as for an established 20% brand in that market (Wellan and Ehrenberg, 1988). Each of the three cases was regarded at the time as unusual and unexpected.

However, a new empirical finding across 22 cases is that average purchase frequencies for successful new brands were in fact normal almost instantly; that is, they were at the Dirichlet level as for any brand in that market (Ehrenberg and Goodhardt, 2000). One instance is in Table 7 for doctors’ prescriptions for the then new antidepressant Prozac (2.3 prescriptions per prescribing doctor). It follows that the previous isolated cases of near-instant loyalty were not

<table>
<thead>
<tr>
<th>Table 6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Market partitioning: unleaded versus leaded gasoline (percent of buyers of X who also bought Y. Typical brands)</strong></td>
</tr>
<tr>
<td><strong>Britain (Quarter I, 1990)</strong></td>
</tr>
<tr>
<td><strong>Who also bought</strong></td>
</tr>
<tr>
<td><strong>Unleaded</strong></td>
</tr>
<tr>
<td><strong>Ledale</strong></td>
</tr>
<tr>
<td><strong>Es</strong></td>
</tr>
<tr>
<td>Buyers of unleaded Esso</td>
</tr>
<tr>
<td>BP</td>
</tr>
<tr>
<td>Mobil</td>
</tr>
<tr>
<td>Gulf</td>
</tr>
<tr>
<td>Buyers of leaded Esso</td>
</tr>
<tr>
<td>BP</td>
</tr>
<tr>
<td>Mobil</td>
</tr>
<tr>
<td>Gulf</td>
</tr>
<tr>
<td>Penetration</td>
</tr>
</tbody>
</table>

**a** Percent buying at least once in the quarter.
exceptional after all. This finding of near-instant loyalty could not have become apparent if the close similarity of loyalty measures for established brands (as in Table 5) had not already been well established.

4.5. Analyzing market dynamics

The steady-state Dirichlet norms have also been used to diagnose dynamic situations where there are marked changes in market shares and/or in total market size. The steady-state norms can then be used to identify and interpret those measures that are still normal and the measures that have in fact changed. Some isolated cases published so far include the following:

- **A short-term trend**—A few years ago, the total sales of U.S. laundry detergents dropped by as much as 15% from one quarter to the next. Repeat buying was tracked in a “conditional trend analysis” (CTA) as a diagnostic tool. This gives predicted repeat-buying norms from QI to QII conditional on each consumer’s QI purchasing level (Goodhardt and Ehrenberg, 1967; Morrison, 1969). Table 8 shows that the sales drop was hardly due to a loss of loyalty (neither light nor heavier QI buyers bought much less than expected in QII). It was mainly due to too few “new” buyers coming in only 25% QI nonbuyers buying in QII rather than the expected 34%.

- **Stockouts**—Temporary out-of-stocks need not lead to a longer-term loss of sales: CTA showed experimentally that before-to-after repeat buying was normal, as if there had been no out-of-stock. Consumers had not learnt to like their “new” brands during the forced switching (Charlton et al., 1972).

- **Extra product promotions**—In an isolated analysis, the giant pack of Unilever’s leading UK laundry detergent was promoted with extra product. Management expected this to appeal to the brand’s heavy buyers. But the CTA norms showed the opposite—the offer of extra product attracted recent nonbuyers and light buyers (Ehrenberg, 1988).

- **The seasonal soup market**—The CTA norms showed that the winter peak of soup sales was about half due to all-year-round buyers buying more and half due to peak-season-only buyers (Wellan and Ehrenberg, 1990). This segmentation would affect the timing of advertising and any attempts to spread peak demand.

- **Price-related promotions**—A wider use of Dirichlet-type norms in analyzing 150 price promotions in Britain, Germany, Japan, and the United States has shown that (i) before-to-after repeat buying is unaffected (i.e., no “learning”), (ii) no before-to-after sales increase occurs, and (iii) there are virtually no new buyers (Ehrenberg et al., 1994a). The conclusion was that price promotions are mainly taken up by existing customers of the brand: promotions just bring forward purchase timing where the brand is already in the purchase repertoires of consumers.

- **Long-term trends**—The 1992 U.S. instant coffee market, as in Tables 1 and 2, had changed radically from 10 years before in two ways: (a) category sales had slumped by two thirds, and (b) Folger’s brand share had doubled. Nonetheless, the loyalty levels were predictably Dirichlet both in 1992 and 1981 (Ehrenberg, 1997). Only the market shares and penetrations had changed. Loyalty was unaffected by the market trends. In another long-run study, the UK toothpaste market was similarly found to be Dirichlet both in 1967/1968 and 1990/1991 despite a major decline in sales of Gibbs and the introduction of successful new brands—Crest, Aquafresh, and Sensodyne (Ehrenberg et al., 1994b).

These various cases seem to suggest that penetration—the number of customers—is the main factor that changes when sales increase. This is strongly supported by a large systematic study, which shows that although penetration and loyalty both grow when share grows (in line with the Dirichlet/DJ cross-sectional results rehearsed here), increased penetration is the key factor (Baldinger et al., 2002). More work into market dynamics is planned.

4.6. Further applications

Dirichlet norms have been used as benchmarks in exploring many other marketing issues. Empirical examples are: (i) cannibalization (Lomax et al., 1996), (ii) price sensitivities (Ehrenberg et al., 1997b; Scriven and Ehrenberg, 1995; Scriven, 1999), (iii) consumer loyalty programs (Sharp and Sharp, 1997, 1999), and (iv) subscription markets (Sharp et al., 2002). Our general conclusion is that the well-grounded steady-state norms reflected by the NBD-

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Table 7
A new brand: near-instant loyalty (doctors’ “new” prescriptions of Prozac)

<table>
<thead>
<tr>
<th>Prozac</th>
<th>Quarters</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(I)(^a)</td>
<td>II</td>
</tr>
<tr>
<td>Percent prescribing</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Average prescription rate(^b)</td>
<td>1.0</td>
<td>2.3</td>
</tr>
</tbody>
</table>

\(^a\) Prozac was launched at some time during this quarter.

\(^b\) Average new prescriptions a quarter per prescribing doctor.

---

Table 8
Conditional trend analysis (CTA): powder detergent

<table>
<thead>
<tr>
<th>A 15% sales drop from QI to QII</th>
<th>Buyers of powder detergent in QI ( O ) (%)</th>
<th>( T ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonbuyers</td>
<td>( O ) (%)</td>
<td>( T ) (%)</td>
</tr>
<tr>
<td>Percent buying powder in QII</td>
<td>25</td>
<td>34</td>
</tr>
</tbody>
</table>

\( O \) = Observed measures; \( T \) = theoretical NBD predictions.
Dirichlet successfully provide benchmarks for an unusually varied range of situations.

5. Marketing implications

The Dirichlet framework also has implications for our broader understanding of consumers, brands, and the marketing mix. These implications follow logically from the preceding patterns, models, and applications, as we now outline.

5.1. Understanding consumers

5.1.1. Polygamous consumers

Over any sequence of purchases, consumers tend to buy several brands with largely steady habitual propensities, at least for the time being. Typically, consumers are polygamous rather than either promiscuous or monogamous (except possibly in "subscription" markets). They usually have several steady partners—a repertoire—with one or two usually being favorites (Hammond, 1997).

For a consumer to be "loyal" to a brand, the Dirichlet model does not presuppose any unique or explicit "commitment" to that brand, nor that the consumer be either a particularly heavy or exclusive buyer of it. Indeed, as noted in Section 2, 100% loyals over any extended sequence of purchases are rare and are light buyers anyway.

Because consumers are generally highly experienced, making a further purchase of a brand would not affect a Dirichlet consumer’s probability of buying the brand (the model’s crucial “zero-order” assumption—see Appendix A). Any “learning” would already have occurred in the past. Although purchase feedback is at times hypothesized in the literature—especially in discussions of choice models (e.g., Seetharaman et al., 1999)—only limited empirical support has been cited for this in steady frequent-purchase markets; and even then, this may have been due to some overlooked nonstationarity rather than real feedback (Shoemaker et al., 1977). The notion that a consumer’s past purchasing behavior or current “state dependence” will invariably change the consumer’s brand choice propensities seems to us to put the cart before the horse. Thus, the experienced consumers’ steady purchase propensities can be thought as the outcome of years of past experience. This is the underlying supposition that leads to the Dirichlet model.

Precisely when a consumer buys the product—e.g., whether this week or next—and which particular repertoire brand is then chosen are both assumed in the model to be as-if-random. This is not to say consumers literally toss mental pennies. Instead, specific choices tend to be governed by a variety of reasons, motives, and feelings—such as out-of-stock situations, promotions, special displays, and consumers’ own diverse habits, needs, moods, the mother-in-law coming, etc. Details are extensively discussed in standard consumer behavior texts and papers (e.g., Engle et al., 1995; McAlister and Pessemier, 1982); in choice modeling (e.g., Horowitz and Louviere, 1995); in qualitative studies (e.g., Fournier and Yao, 1997; Gordon, 1994); and in the context of limited problem solving (e.g., East, 1997; Foxall and Goldsmith, 1994; Olshavsky and Granbois, 1979). However, the varying behavior of consumers is sufficiently idiosyncratic and irregular to be successfully modeled mathematically as being quasi-random, especially collectively.

5.1.2. Brand segmentation

Consumers are widely expected to fall into relatively homogeneous and recognizable subgroupings. But such brand segmentation is not directly allowed for in the Dirichlet framework, which nonetheless successfully predicts most BPMs.

Indeed, the vast segmentation literature is surprisingly lacking in explicit empirical cases where directly competitive brands do appeal to different kinds of people. Much of this literature is focused on techniques, not empirical results (e.g., Wedel and Kamakura, 2000). In contrast, there is now much systematic empirical evidence that the user profiles of substitutable brands seldom differ (Collins, 1971; Hammond et al., 1996; Kennedy and Ehrenberg, 2000). In practice, the customers of similar brands are very similar, as would tend to follow if nearly each of them uses several brands.

Nor do brand users generally differ in their attitudes to the brands they buy. For Dirichlet-type markets, the data show that buyers of Brand A feel about A much as buyers of Brand B feel about B (Barwise and Ehrenberg, 1985; Dall’Olmo Riley et al., 1997; Franzen, 1994). This similarity of brand-users’ attitudes is consistent with the small role brands play when consumers actually consume the product: in offering a coffee, the common questions are “milk or sugar?” or perhaps “decaffeinated or regular?” but not the choice of brand “Maxwell House or Nescafé?” Nevertheless, consumers might come to identify with their habitual brands (“I use it, therefore I like it”)—see Barnard and Ehrenberg, 2000).

Despite this lack of segmentation between brands, there can be strong segmentation at the category or subcategory level. Here, consumer habits, needs, and customer types often differ (e.g., Day et al., 1979; Ehrenberg, 1959, 1988; Bock and Uncles, 2002). For example, larger households are usually heavier buyers of the product as a whole. In addition, customers of functionally distinct subcategories differ: cat food buyers have cats and dog food buyers have dogs; users of leaded gasoline had older cars; and presweetened cereals are eaten more by children. But such subpatterns then tend to hold about equally for all the substitutable brands in that category or subcategory.

5.2. Understanding brands

The Dirichlet model is a theory about choice between competitive entities such as brands. Such brands can be near
identical in the theory itself—no differentiating attributes need to be specified except for the brand names and market shares. This is often so in practice since sales-effective product advantages and innovations are usually soon copied (e.g., Ehrenberg et al., 1997a; Foxall, 1999). Even early mover advantages and innovations are usually soon copied shares. This is often so in practice since sales-effective need to be specified except for the brand names and market identical in the theory itself—no differentiating attributes advertised, and some may appear almost meaningless (Car-
competing brands occur in practice, such as different bottle and variants come).

countries will hardly know from which company the brands always—for Unilever and P&G brands, consumers in most 
logg’s or Heinz for all of the variants, although not names (possibly with the same house name such as Kel-

Sometimes these product variants are given distinct brand 
then each launch product variants into these submarkets. 

In many product categories, some functionally distinct submarkets or partitions have been deliberately created (e.g., decaffeinated and regular coffee, luxury cars, unleaded and leaded gasoline). In most instances, direct competitors will then each launch product variants into these submarkets. 

5.3. The role of marketing-mix factors

A major lesson of the Dirichlet-type findings is that brands often differ little in their loyalty-related measures but vary greatly in their penetrations. This means that varying marketing-mix inputs such as changes in price, product formulation, selling, and distribution can have little if any impact on increasing loyalty, but they may affect the
brand’s penetration, and in particular its market share and
sales volume.

The observed outcomes of price-related promotions
support this view, as noted earlier. Large short-term sales
blips (often 100% up or more) show that promotions
have a dramatically ability to attract extra buyers. But the
blip stems almost entirely from past customers—these
only need to change their quasi-random purchase timing
of the promoted brand rather than more fundamentally
switch to an unfamiliar brand (Ehrenberg et al., 1994a).
Hence, it need not be surprising that promotional sales
blips last only while “giving the product away” and do
not affect repeat buying loyalty thereafter (Abraham and
Lodish, 1987; Jedidi et al., 1999).

For brand advertising, the bulk of it occurs between
competitive brands, between which there is usually little
sustainable differentiation. Hence, advertising does not, we
believe, have to try to persuade consumers that Brand A
really differs from the similar B, or even that it is better than
B. Consumers just have to choose one of the available
options of the right type: “Any reasonable brand will do”
(Heath, 1999). In practice, consumers seem then to find it
convenient and reassuring to have already developed their
habitual split-loyalty choice propensities. Advertising an
established brand therefore must mainly publicize the brand,
mostly by reminding experienced consumers—“Here I am,
remember me”—often in highly creative ways for impact
and memorability (Ehrenberg, 1974; Barnard and Ehren-
berg, 1997; Ehrenberg et al., 2002).

5.4. Marketing management issues

The ubiquity of the predictable Dirichlet-type patterns of
buying, together with most markets being more or less
steady most of the time, could add to existing doubts about
the role of marketing. For example, are marketing inputs
such as advertising worth it if they do not soon lead to extra
sales? Our view is that there is nonetheless plenty of scope
for marketing (e.g., securing better distribution) since mar-
ket shares differ greatly. However, such scope exists also for
competitors. The outcome is therefore usually competitive
equilibrium rather than actual big gains or losses. This calls
for maintenance of sales and retention of split-loyalty
customers (both consumer and trade). In this sense, com-
petition means running hard to stand still, with profitable
survival being preferable to the most likely alternative.
Any dramatic and lasting gain is a rare bonus.

6. Conclusion

To conclude, the Dirichlet model describes the widely
observed patterns of near-steady state buying behavior
that tend to occur. In this, the model consistently predicts
fairly complex patterns and BPMs from very simple and
limited inputs. It also helps to explain the patterns;
namely, that under near-steady state conditions, traditional
loyalty-related measures for competitive brands are unaf-
fected by any marketing-mix inputs, other than indirectly
through changes in the brand’s market share.

While the Dirichlet model itself is explicitly defined for
steady-state and nonpartitioned zero-order markets, it in no
way implies or predicts that all markets should be near
stationary and nonpartitioned, as we have already stressed.
The model merely describes what markets are like when they
are more or less steady and nonpartitioned with no “purchase
feedback.” But since many markets or submarkets are like
that most of the time, it behooves the management to try and
understand such steady markets and the factors that deter-
mine their sales. These factors are how many customers a
brand has, how loyal they are, which competitive brands
customers also buy, and how often they do so, together with
the Dirichlet type of constraints on all this.

An analogy for the prime importance of the near-steady
state in the Dirichlet view of markets lies in our use of
automobiles. Dynamics matter: we have to start and stop.
But driving at more or less steady speeds accounts for most
of the distance covered. Similarly, near-steady state markets
generate the bulk of our sales and therefore they are likely to
account for most of our revenue.

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Appendix A. The Dirichlet theory

We discuss: (1) the theoretical assumptions of the Dirich-
let, (2) how the model is calibrated, (3) how the theoretical
performance measures are estimated, and (4) alternative
models of buyer behavior.

A.1. The NBD-Dirichlet assumptions

In the NBD-Dirichlet model (or “Dirichlet” for short),
consumers are seen to have, for the time being, steady
personal purchase propensities—or stochastic probabili-
ties—for when they buy the product and what brands they
then choose. This involves five distributional assumptions
(Goodhardt et al., 1984). Two assumptions are about buying
the product category:

(i) A Gamma distribution for consumers’ differing average
purchase rates
Each consumer is assumed to buy the category at some
steady long-run rate (e.g., once a year or 10 times a
A given consumer’s specific purchases of the category are assumed to be spread irregularly (“as-if-randomly”) over time with each consumer’s long-run probability (e.g., about 0.02 or 0.20) and independently of when the previous purchase was made (a “zero-order” process). This specifies “Poisson” distributions for each consumer, as has been widely tested for brand purchasing (Bass et al., 1984; Chatfield and Goodhardt, 1973; Dunn et al., 1983; Schmittlein et al., 1985). The zero-order assumption makes simple sense with the typically low weekly purchase probabilities: even for someone buying 10 times a year, the chance of buying in two successive weeks is only 0.04 and hence near zero—no special assumption of “dead periods” between purchases has to be made.

The Poisson and Gamma assumptions combine to give the negative binomial distribution or “NBD” model (Ehrenberg, 1959). The NBD has been used widely in repeat-buying studies where each brand is considered on its own. In the multibrand Dirichlet model, the NBD is used for the category distribution of purchases.

The third and fourth Dirichlet assumptions are about brand choice:

(iii) A multivariate Beta distribution of brand choice probabilities.

Heterogeneity in consumers’ brand choice probabilities is assumed to follow a smooth Beta distribution of a multivariate “Dirichlet” type (after a French-named German mathematician). Strong theoretical backing for this assumption derives from the observed near independence of brand choice in line with the traditional theoretical notion of “independence from irrelevant alternatives” or IIA (Goodhardt et al., 1984; Leeflang et al., 2000; Luce, 1959; Morrison and Schmittlein, 1988).

(iv) Multinomial distributions for specific purchases

On any one category purchase occasion, a consumer is assumed to choose a brand as if randomly with their own fixed brand choice probabilities. This is the widely used zero-order “multinomial” distribution of brand choice (typically, consumers have “reasons” but behave as-if-random with their personal choice probabilities). Crucially, the probability of buying Brand A on a particular occasion is independent (“zero order”) of the brands the consumer has previously bought. Thus, a consumer with a three-brand portfolio and zero-order probabilities of 0.6, 0.3, and 0.1 will buy the three brands 60%, 30%, and 10% of the time in a random order.

An equivalent specification is used in choice models (e.g., Luce, 1959; Ben-Akiva and Lerman, 1985; McFadden, 1986). The deterministic part of the choice model might suggest that utility for Brand A is higher than the utility for other brands, but the random error terms in the utilities can result in a consumer choosing a brand other than A. Thus, if the deterministic part of the utilities are in the ratio of 0.6, 0.3, and 0.1 for a three-brand portfolio, a consumer does not always choose the brand with a utility of 0.6. In aggregate, the three brands are bought 60%, 30%, and 10% of the time.

The fifth Dirichlet assumption is about the relationship between product category buying and brand choice:

(v) The independence of purchase incidence and brand choice.

The Beta distributions of brand choice probabilities are taken to be the same irrespective of how often particular consumers buy. This is in line with observed market shares being much the same for light, medium, and heavier category buyers.

In summary, the Dirichlet model specifies how many purchases each household (or individual consumer) makes of each of the available (or specified) g brands in a chosen period of length T (not the T for the theoretical Dirichlet predictions). The model represents this as a g-variate discrete random variable with a joint frequency distribution given by the mixture of the Multinomial, Dirichlet, Poisson, and Gamma distributions (e.g., Ehrenberg, 1988, p.258; Rungie, 1999).

Having specified the model by these distributional assumptions, the parameters of the model must next be numerically calibrated. Then any brand’s theoretical performance measures have to be estimated. This we now describe in two stages.

A.2. Calibrating the model

To calibrate the Dirichlet model for a given product or service category, four observed input measures are used, two penetrations and two purchase rates, as is shown in Table A1.
For some chosen “base period” (which will be greater than the average interpurchase interval), such as a year, the inputs are as follows:

- \( B \), the category penetration (i.e., households who bought the category of instant coffee at least once in the year, 31%).
- \( b_F \), the penetration of one particular brand, Folgers say (11%).
- \( W \), the average frequency of buying the category per category buyer (5.0).
- \( w_F \), the average frequency of buying Folgers (3.2).

(We note that penetrations are usually expressed as proportions algebraically, but as percent numerically).

If the model fits the observed data exactly, any brand could be used in this calibration process and would give the same results. In practice, there are deviations. It is then usual to fit the model for each of a number of leading brands and to average or smooth the results (in the computer program). The selection of brands to use in such model calibration can be deliberate. In particular, any unusual brand can be left out of the calibration process (e.g., the unusual Maxim in Table 1 or brands with abnormal availability like a private label or regional brand). But their estimated performance measures would still be checked subsequently. The mathematics of this calibration process are fairly complex, but software is available (e.g., Uncles, 1989; Hewitt, 1990; Kearns, 2000; Rungie et al., 2003).

### Table A1
The inputs for calibrating the Dirichlet parameters (as required for Tables 1, 2, and 5)

<table>
<thead>
<tr>
<th>Instant coffee (USA 1992)</th>
<th>Market share</th>
<th>Percent buying in year</th>
<th>Purchases per buyer</th>
<th>Percent buying 5+ times</th>
<th>Category purchases per buyer</th>
<th>100% loyal</th>
<th>Percentage who also bought:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( O )</td>
<td>( T )</td>
<td>( O )</td>
<td>( T )</td>
<td>( O )</td>
<td>( T )</td>
</tr>
<tr>
<td>Category</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Folgers</td>
<td>24*</td>
<td>31</td>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maxwell House</td>
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</tr>
<tr>
<td>Tasters Choice</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nescafe</td>
<td>11</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Sanka</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High Point</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maxim</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brim</td>
<td>0.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Average Brand</strong></td>
<td></td>
<td>31</td>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* A brand’s market share (or equivalent) is needed to estimate its theoretical performance measures.

A method based on using the Joster scale to replace panel data has been described too (Wright et al., 2002).

### A.4. Simplifying approximations

Alternatively, simple verbal descriptions can be used to communicate what the theoretical estimators say—like those in Section 2 for the observed patterns in the data. Thus, for “DJ,” we said that far fewer people buy a small brand and buy it somewhat less often (this is like the wording commonly used to describe the normal distribution are aggregated in the model to give a theoretical estimate or “prediction” of any chosen performance measure, such as a particular brand’s penetration or its average purchase frequency.

The algebra for the model’s theoretical estimate of a chosen performance measure is again complex, as is illustrated in Table A2. In practice, one can use computer software (contained in the same packages as for the model calibration) to give numerical estimates such as those in Tables 1 and 5 earlier. It is also possible to simulate individual Dirichel-type consumers and then tabulate any required measure, as one would with observed data (Goodhardt, 1995). This can be useful for theoretical measures, which are not covered by the available estimation software.

### Table A2
The Dirichlet’s theoretical formulae for the penetration

<table>
<thead>
<tr>
<th>Brand</th>
<th>( S_{\alpha} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Folgers</td>
<td>[ C_{n} = C_{n-1} \frac{K + n - 1}{n} \frac{S(1 - z_{\alpha}) + n - 1}{(S + n - 1)} \frac{A}{1 + A} ]</td>
</tr>
<tr>
<td>Nescafe</td>
<td></td>
</tr>
<tr>
<td>Maxim</td>
<td></td>
</tr>
</tbody>
</table>

where \( K, A, \) and \( S \) are the parameters of the fitted Dirichlet model.
in statistics, where we say “5% lie beyond ±2 standard deviations” rather than spell out the pdf).

Furthermore, several close quantitative approximations to the complex Dirichlet-type estimators are also available. These are much simpler than the exact Dirichlet expressions and therefore more insightful. The two most used simplifications are: (i) the “DJ” relation \( w_x(1-b_x) \propto w_o \), a constant for that set of brands and chosen time period (this relates Brand X’s average purchase frequency \( w_x \) directly to its penetration \( b_x \)), and (ii) the “Duplication Law” relation \( b_{xy} \propto Db_x \) (this relates the proportion \( b_{xy} \) of buyers of Brand Y who also buy Brand X in the analysis period, to X’s overall penetration \( b_x \), and where \( D \) is defined below).

### A.4.1. The “\( w(1-b)=constant \)” approximation

This says that \( w \) varies with \( 1/(1-b) \). Thus, the smaller the \( b \) (as a proportion), the smaller is \( 1/(1-b) \) and hence the smaller is \( w \). This is McPhee’s DJ notion that was noted earlier (but is not readily apparent from the exact but complex Dirichlet-type algebra such as in Table A2 where \( w_x \) and \( b_{xy} \) would depend on all the observations for all the brands and not just on \( b_x \) (Ehrenberg, 1988).

The approximate formula predicts well, as illustrated in the final column of Table A3. For brands with fairly low penetration, the average purchase rates hardly differ (e.g., for \( b_{xy} \) of 10% or less as in Table A3, the \( w_x \)s are all about 3). Only for brands with high penetration are the differences in average purchase frequencies quite large. Relatively high penetrations can occur both for large brands, and for small ones over long time-periods. Remarkably, the \( w(1-b) \) formula copes with both cases.

### A.4.2. The “\( b_{xy}=Db_x \)” approximation

This says that \( b_{xy} \), the percentage of buyers of Brand Y who also buy X in the chosen analysis-period, is simply proportional to \( b_x \), Brand X’s penetration in the whole population (e.g., Ehrenberg and Goodhardt, 1968b, 1970; Ehrenberg, 1988; Goodhardt, 1966). This approximate relationship, or “duplication of purchase law,” is illustrated in Table A4 for the duplication of purchase of Folgers with the other instant coffee brands. The duplication \( D \) is very simple to estimate as the average of the observed duplications for all pairs of brands (leaving out any extreme outlier) divided by their average penetration.

Expressing brand penetration in “relative” terms as the percentage of category buyers who buy the brand (i.e., \( b/B \)) probably leads to further simplifications. Thus, the duplication coefficients \( D \) would tend to be close to 1 (implying independent brand choice, a very simple result). But the base \( B \) then varies with the length of the analysis period. More work is needed.

### A.5. Elaborating the Dirichlet model

The Dirichlet and related stochastic approaches to modeling buyer behavior have been elaborated in two main ways. First, case-specific “mass points” have been used to model consumer heterogeneity rather than the Dirichlet’s smooth Gamma and Beta distributions (e.g., Colombo and Morrison, 1989; Reader and Uncles, 1988; Fader and Hardie, 1996). In a one-off study of saltine crackers, a semiparametric random effects model was found to capture heterogeneity in brand preferences across households, but the predictive gains were slight and came at the expense of added complexity (Chintagunta et al., 1991). It appears that with these alternative models, there is some scope to improve goodness of fit, but only by forsaking parsimony and generalizability. Nor so far have these alternatives reported the same range of BPMs that can be derived routinely from the Dirichlet.

Second, consumer heterogeneity has been related to various possible causal sources such as sociodemographic and attitudinal factors (Allenby and Lenk, 1994; Bass, 1993; Bhattacharya et al., 1996; Ehrenberg, 1959; Fader, 1993; Fader and Lattin, 1993; Jones and Zufryden, 1980; Russell
and Kamakura, 1994; Vilcassim and Jain, 1991; Wrigley and Dunn, 1985). A variety of factors have proven to be statistically significant in isolated studies, but systematic effects and generalizations have eluded analysts. Moreover, few major gains in predictive power have been either reported or even claimed for these more elaborate model specifications.

A.6. Other modeling approaches

The Dirichlet model, and its underlying descriptive and benchmarking uses, differs from many other modeling approaches. The alternative stochastic and/or econometric response models for buyer behavior in the literature are mostly dynamic. Many aim to predict changes in market shares from changing marketing-mix inputs (e.g., advertising or price). In this, they are generally attempting to model nonstationary market conditions (e.g., Guadagni and Little, 1983; Leeflang et al., 2000; Lenk et al., 1993; Kannan and Yim, 2001; Vilcassim and Jain, 1991; Wagner and Taudes, 1986; Yim and Kannan, 1999).

The parameters of such econometric-type models are mostly respecified on each occasion. Some of the models assume turbulent (i.e., continually changing) choice probabilities for each consumer (e.g., Erdem, 1996; Erdem and Keane, 1996). Such models usually require many—possibly hundreds—of parameters to be estimated and interpreted, with conceptual and at times computational complexities (e.g., multicollinearity), which seem largely unresolved. Specific results have been reported but with few generalizable findings and insights.

Economists and psychologists such as Kahneman and Tversky have carried out very innovative research into broader issues of consumer choice, often along experimental and/or mathematical lines and frequently with roots in game theory (e.g., Kagel and Roth, 1995; Thaler, 1994). But they seldom touch on the issues of brand choice in highly competitive markets, covered by standard scanner panel tabulations and the Dirichlet approach.

Nearly all such modeling appears to be aimed at discovering and predicting effects that have not yet been observed. Indeed, these other approaches tend not to mention any of the ubiquitous empirical patterns reported in Section 2 or their theoretical modeling as described in Section 3 (see for example reviews by Bucklin and Gupta, 1999; Hanssens et al., 1990; Leeflang and Wittink, 2000; Lilien et al., 1992). Even the early theoretical derivation of the Dirichlet model by Bass et al. (1976) did not refer to any empirical patterns. In contrast, the Dirichlet-type approach first establishes what kinds of empirical changes have in fact taken place and then tries to model that knowledge (Ehrenberg, 1993b).

For example, do any sales increases for a given brand follow the steady-state DJ pattern for large and small brands (implying mainly penetration growth rather than separate changes in buying rates or loyalty)? Do sales decreases simply show the reverse pattern from sales increases or are they more complex? Do increased sales for Brand X come from competitive brands pro rata to their size (as in the steady-state duplication of purchase law), or do gains and losses operate quite differently from that? These are empirical questions for which it seems to us useful to know whether some answers generalize before theorizing extensively about them.

A.6.1. Other structural models

The Dirichlet model is purely structural for steady-state markets, as we have stressed, with no explicit “explanatory” variables specified (e.g., advertising or price changes). An alternative structural model is the first-order Markov process. This was popular in the 1960s and 1970s (e.g., Kuehn, 1962; Massy et al., 1970) and has occasionally been mentioned ever since (e.g., Bronnenberg, 1998; Leeflang et al., 2000). In its simplest form, the Markov approach also has no “causal” inputs but assumes (i) homogeneous consumers and (ii) fixed switching and repeat-purchase probabilities for each brand. These assumptions run counter to the Dirichlet-type empirical findings (and to much of the response modeling literature more generally). It is now known that (i) consumers are heterogeneous and (ii) that switching and repeat buying are independent of the brands as such but vary with market shares. It is seldom that a modeling theory and the facts clash so starkly.

Another purely structural model is the Hendry system (Butler, 1966) and the derivatives of it (e.g., Kannan and Sanchez, 1994). This uses the Duplication of Purchase Law formulation for pairs of purchases. But it makes very different assumptions from the Dirichlet and has different outcomes (e.g., Ehrenberg and Goodhardt, 1974). More generally, “pairs of purchases” can however be used very constructively in the absence of continuous panel data (e.g., Bennett, 2002; Colombo et al., 2000; Ehrenberg and Bound, 2000).

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